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Boundary velocity method for continuum shape sensitivity of nonlinear fluid–structure interaction problems ☆

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ABSTRACT

A Continuum Sensitivity Equation (CSE) method was developed in local derivative form for fluid–structure shape design problems. The boundary velocity method was used to derive the continuum sensitivity equations and sensitivity boundary conditions in local derivative form for a built-up joined beam structure under transient aerodynamic loads. For nonlinear problems, when the Newton–Raphson method is used, the tangent stiffness matrix yields the desired sensitivity coefficient matrix for solving the linear sensitivity equations in the Galerkin finite element formulation. For built-up structures with strain discontinuity, the local sensitivity variables are not continuous at the joints, requiring special treatment to assemble the elemental local sensitivities and the generalized force vector. The coupled fluid–structure physics and continuum sensitivity equations for gust response of a nonlinear joined beam with an airfoil model were posed and solved. The results were compared to the results obtained by finite difference (FD) method.

1. Introduction

In the continuum sensitivity equation (CSE) method, also known as variational shape design (Haug et al., 1986), the variational sensitivity method (Haftka and Gurdal, 1992), and the continuous sensitivity equation method (Borggaard and Burns, 1997), the design parameter gradients are calculated by solving the continuum sensitivity equations. Since the CSE system in local derivative form is posed as a continuous system, it can efficiently produce shape parameter gradients without calculating the mesh sensitivity inside the discrete domain. Further, the sensitivity system is always a linear system of equations, even when the analysis problem is nonlinear. If Newton–Raphson iteration is used for nonlinear problems, the tangent stiffness matrix gives the sensitivity coefficient matrix for the CSE linear system as previous researchers, Borggaard and Burns (1994), and Wickert (2009), have noted.

CSE methods were first introduced for structural problems (Dems and Mroz, 1985; Haftka and Gurdal, 1992). Jameson (1988) first introduced the continuum sensitivity concept in adjoint equation form for aerodynamic design problems. Borggaard and Burns (1994) introduced the CSE nomenclature in a fluid setting and several fluid flow optimization applications followed (Bhaskaran and Berkooz, 1997; Borggaard and Burns, 1997; Stanley and Stewart, 2002; Turgeon et al., 1999). Choi and Kim (2005) cites the lion's share of structural elasticity applications that employ continuum sensitivity

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methods. Bhaskaran and Berkooz (1997) present one of the earliest uses of CSE for a fluid–structure interaction problem, although it involved CSE only for the fluid with respect to structural mode shapes for the purpose of creating an aerodynamic influence coefficient matrix for flutter calculation. Pelletier et al. have employed CSE methods for a range of fluid–structure interaction (FSI) problems with prominent success (Etienne and Pelletier, 2005; Etienne et al., 2006, 2007); however, much of their work focuses on flow sensitivities. Wickert and Canfield recently applied the CSE method to the gust sensitivity for an airfoil mounted to an Euler Bernoulli beam (Wickert et al., 2009). They used least-squares finite elements and solved the nonlinear system with successive substitution for which the nonlinear coefficient matrix is different than the sensitivity matrix. Recent work has applied the CSE method to built-up structures, such as joined beam structures, with only static results (Liu et al., 2010). The current effort extends this method to a transient nonlinear joined beam with an airfoil, using the Galerkin finite element method. The Newton–Raphson method is used for the nonlinear problem and for obtaining the sensitivity coefficient matrix for the linear CSE system. Boundary conditions are also derived for CSE of built-up structure sensitivity problems.

CSE systems are often posed in terms of local derivatives (an Eulerian reference frame) for fluid flow, although it is possible to derive the fluid CSE system in total derivative form (Lagrangian reference frame) (Charlot et al., 2009, 2012). For shape optimization of fluid applications, an Eulerian description of the flow sensitivity is often adequate, once accuracy of the spatial derivatives of the solution on the boundary are addressed, since often only local sensitivities for flow are of interest. Thus, much of the fluid mechanics literature does not emphasize the distinction between the local and total derivative. For structural optimization problems, however, the design sensitivity at a material point is usually required, which necessitates a means to calculate total sensitivities at a given material point. In this work, local derivative form CSE system is solved and then the local derivatives are transformed to the total derivative when needed. The difficulty of implementing the local derivative form for structures with stress discontinuity, such as built-up structures, is pointed out and a remedy is suggested.

Section 2 begins with a derivation of the CSE system and its associated boundary conditions. In the next section, CSE boundary conditions at the joint of a built-up structure are discussed. Section 4 gives the finite element model and the local form CSE for both structure and fluid domains of the nonlinear joined beam and airfoil model. The model is fairly simple, yet complex enough to capture all the salient aspects of the CSE method for a coupled domain, nonlinear, transient system. This is followed by a brief description of the computational results for a simplified nonlinear beam with potential flow about an airfoil model and a joined beam structure under static and transient aerodynamic load.

2. Continuum sensitivity equations

Consider the following general, nonlinear boundary value system defined in a domain Ω with a boundary Γ for which we seek a solution $u(x,t;b)$ of the

$$\mathcal{A}(u, L(u)) = f(x,t;b) \quad \text{on } \Omega, \tag{1}$$

with geometric and natural boundary conditions:

$$\mathcal{B}(u, L(u)) = g(x,t;b) \quad \text{on } \Gamma, \tag{2}$$

where $u = u(x,t;b)$ is dependent on design variable b implicitly, L is a linear differential operator, such as $\{\partial/\partial t, \partial/\partial x, \partial/\partial y, \partial^2/\partial x^2, \partial^2/\partial y^2 \ldots\}$, that appears in the governing differential equations or boundary conditions, $\mathcal{A}$ and $\mathcal{B}$ are vectors of algebraic functions of u and $L(u)$, and $\mathcal{B}(u, L(u))$ can be a simple function of u, such as a prescribed boundary condition $u = \overline{u}$ for Dirichlet boundary conditions, or involve a differential operator for von Neumann boundary conditions.

Discrete sensitivity analysis methods discretize the original governing equation (1) and obtain a system of discrete equations first. For example, in the static case

$$\mathbf{Ku} = \mathbf{f}. \tag{3}$$

Then, the sensitivity with respect to the design parameters, $D\mathbf{u}/Db$, can be calculated by using the finite difference method or discrete analytical methods.

The total derivative of solution vector $\mathbf{u}$ with respect to design parameter b is $D\mathbf{u}/Db$. The finite difference method can be used for approximating $D\mathbf{u}/Db$. For example, the forward difference approximation is

$$\frac{D\mathbf{u}}{Db} \simeq \frac{\mathbf{u}(b+\Delta b) - \mathbf{u}(b)}{\Delta b}. \tag{4}$$

A drawback of the finite difference method is the challenge of determining the optimum step size. Large step sizes are dominated by truncation error and small step sizes are dominated by numerical round-off error. Furthermore, it involves solving the original analysis problem $n+1$ times for n design variables, which makes finite difference method inefficient, especially for nonlinear problems.

Discrete analytical methods require the derivatives of the stiffness matrix and load vectors with respect to the design variables. The direct method differentiates Eq. (3) and solves for

$$\frac{D\mathbf{u}}{Db} = \mathbf{K}^{-1}\left(\frac{D\mathbf{f}}{Db} - \frac{D\mathbf{K}}{Db}\mathbf{u}\right), \tag{5}$$