MATERIALS CHARACTERIZATION 84 (2013) 67–71

Available online at www.sciencedirect.com

SciVerse ScienceDirect

www.elsevier.com/locate/matchar

Notes on representing grain size distributions obtained by electron backscatter diffraction

Laszlo S. Toth[a,b], Somjeet Biswas[b], Chengfan Gu[c,*], Benoit Beausir[a,b]

[a]Laboratoire d'Etude des Microstructures et de Mécanique des Matériaux (LEM3), UMR 7239, CNRS/Université de Lorraine, F-57045 Metz, France

[b]Laboratory of Excellence on Design of Alloy Metals for low-mAss Structures (DAMAS), Université de Lorraine, France

[c]School of Materials Science and Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

A R T I C L E D A T A

Article history:
Received 29 April 2013
Received in revised form 13 July 2013
Accepted 19 July 2013

Keywords:
Grain size distribution
Number fraction
Area fraction
Density function
EBSD

A B S T R A C T

Grain size distributions measured by electron backscatter diffraction are commonly represented by histograms using either number or area fraction definitions. It is shown here that they should be presented in forms of density distribution functions for direct quantitative comparisons between different measurements. Here we make an interpretation of the frequently seen parabolic tales of the area distributions of bimodal grain structures and a transformation formula between the two distributions are given in this paper.

1. Introduction and Definitions

Electron Backscatter Diffraction (EBSD) is a powerful tool to study microstructures performing quantitative metallography [1]. One of the most important characteristics of microstructures of polycrystalline materials is its grain size distribution. The results of measurements are usually presented in histograms where the number of grains with their diameter D_i lying in an interval δD_i is counted as a function of grain size. δD_i is called a 'bin' and it is practical to chose the bin-size constant, thus, in the following, the index i is only used to identify a bin, it does not involve differences in size of the bin. Two kinds of representations are currently in use for 2D surfaces; the number fraction [2–4] and the area fraction definitions [5,6]. Even if applied for the same measurement, the two representations are quite different.

The number-fraction distribution $F_N(D_i)$ is defined by

$$N_i = F_N(D_i)N_{total} \quad (1)$$

where N_i is the number of grains in δD_i and N_{total} is the total number of grains. Similarly, the area fraction distribution $F_A(D_i)$ defines the sum of the areas A_i of grains that have their diameter D_i in the bin zone:

$$A_i = F_A(D_i)A_{total}. \quad (2)$$

Here A_{total} is the total area occupied by the grain, it is equal to the surface of the measurement if all pixels belong to grains in EBSD. It follows from the definitions above that the sum of

* Corresponding author.
E-mail addresses: laszlo.metz@univ-lorraine.fr (L.S. Toth), somjeetbiswas@gmail.com (S. Biswas), chengfan.gu@unsw.edu.au (C. Gu), benoit.beausir@univ-lorraine.fr (B. Beausir).

1044-5803/$ – see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.matchar.2013.07.013

the fraction-values added up for all the bins δD_i is equal to one (or 100% in percentage presentation):

$$\sum_{i=1}^{n} F_N(D_i) = 1, \quad \sum_{i=1}^{n} F_A(D_i) = 1. \tag{3a, b}$$

Here n is the total number of bins along the grains size axis.

Either using number-fraction or area-fraction distributions, the values of $F_N(D_i)$ and $F_A(D_i)$ depend on the length of the chosen bins δD_i which makes the results obtained with different bin incompatible to each other so any quantitative comparison of them is difficult. This incompatibility can be resolved by using normalized density distribution functions for both types of distributions:

$$f_N(D_i) = \frac{F_N(D_i)}{\delta D_i}, \quad f_A(D_i) = \frac{F_A(D_i)}{\delta D_i}. \tag{4a, b}$$

$f_N(D_i)$ and $f_A(D_i)$ are called density distribution functions. Then the following summations are satisfied:

$$\sum_{i=1}^{n} f_N(D_i)\delta D_i = 1, \quad \sum_{i=1}^{n} f_A(D_i)\delta D_i = 1. \tag{5a, b}$$

These density distribution functions can be continuous if $\delta D_i \to 0$ and the summations in Eq. (5a,b) become integrals:

$$\int_{d_{\min}}^{d_{\max}} f_N(D)dD = 1, \quad \int_{d_{\min}}^{d_{\max}} f_A(D)dD = 1. \tag{6a, b}$$

The meaning of the number-fraction density $f_N(D_i)$ and area-fraction density $f_A(D_i)$ distribution functions is as follows:

$$f_N(D_i)\delta D_i = \frac{N_i}{N_{total}}, \quad f_A(D_i)\delta D_i = \frac{A_i}{A_{total}}. \tag{7a, b}$$

The subject of the present work is to compare the two kinds of functions, to give a transformation formula between them and to explain a particular property of the area density distribution usually observed for bimodal grain structures. An experimental example is taken from EBSD maps measured after dynamic recrystallization of a Mg alloy in torsion (AM30, see more about the experiments in [7]).

2. Comparison of Number and Area-weighted Density Distributions

Fig. 1 shows the microstructure of a Mg AM30 alloy obtained by EBSD after torsion at 250 °C to a shear strain of 1.73 [7]. Due to partial dynamic recrystallization, there is a large population of small grains and another population of non-recrystallized grains in the measurement. The grain size distribution is displayed in Fig. 2 for both number and area density functions. The bimodal nature of the distribution appears in the area density representation as a large peak at small grain sizes and a nearly uniform distribution for grain sizes larger than about 10 μm (Fig. 2a). In the number-density representation there is only one peak because the small grains very much outnumber the large ones (Fig. 2b). The advantage of showing the area density distribution is clear for bimodal structures.

For larger grains the area-density distribution has some interesting features because the intensity values are situated along some parabolic functions. First we make a quantitative description of these parabolic parts of the distribution. Let us consider such bins in the measurement in which there are the same number of grains denoted by n^*. The smallest number of n^* is 0 for which case there is no vertical bar in the histogram. For the case of area-density distribution, the total surface area of the grains with diameter values lying in a given bin can be approximated by using the equivalent circle area method:

$$A_i \cong \frac{n^* D_i^2 \pi}{4}. \tag{8}$$

Here D_i is the diameter value in the middle of the interval δD_i. Using this expression in Eq. (7a,b), we obtain the equation of such specific distribution:

$$f_A^{n*}(D_i) = n^* \frac{\pi}{4A_{total}\delta D_i} D_i^2. \tag{9}$$

$f_A^{n^*}$ is the distribution function of those grains for which there is the same number n* grain in a bin. As we can see, this is a simple parabolic function. Fig. 2a displays $f_A^{n^*}$ for increasing value of n^*. The large grain size part of f_A can be perfectly described by $f_A^{n^*}$ for $n^* = 1$, meaning that there is a maximum of only one grain in a bin. For increasing value of n^* there are also several parts of f_A which can be well described by $f_A^{n^*}$. Due to the nature of the construction of the histogram, any value in the distribution corresponds to a certain n^*, so the whole distribution can be described by a set of $f_A^{n^*}$ functions with varying n*.

The function equivalent to $f_A^{n^*}$ in the number-density distribution is just a horizontal line defined by $N_i = n^*$ so we obtain using Eq. (7a,b):

$$f_N^{n*} = \frac{n^*}{N_{total}\delta D}. \tag{10}$$

(Here the subscript i is dropped from δD_i because δD_i is constant.) These horizontal lines are shown in the inset of Fig. 2b for large grain sizes. The largest value of n^* necessarily corresponds to the maximum value for the f_N distribution. However, this is not the case for the f_A distribution. The position of the maximum n* in f_A is obtained from Eq. (9):

$$n^*_{\max} = \max\left[\frac{4A_{total}\delta D_i f_A(D_i)}{\pi D_i^2}\right]. \tag{11}$$

The position corresponding to Eq. (11) is identified in Fig. 2a, located at D_i = 3.13 μm with an $n^*_{\max}$ value of 1294. It is important to know that the maximum value does not correspond to the place which is most populated by the grains in an area-density distribution.

The average grain size is an important information that characterizes the microstructure. It is defined as follows:

$$\overline{D}_N = \frac{\sum_{i=1}^{n} f_N(D_i)D_i\delta D_i}{\sum_{i=1}^{n} f_N(D_i)\delta D_i} = \sum_{i=1}^{n} f_N(D_i)D_i\delta D_i = \sum_{i=1}^{n} F_N(D_i)D_i, \tag{12}$$