Journal of Fluids and Structures 27 (2011) 134–144

Contents lists available at ScienceDirect

Journal of Fluids and Structures

journal homepage: www.elsevier.com/locate/jfs

Nonlinear and viscous effects on wave propagation in an elastic axisymmetric vessel

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ARTICLE INFO

Article history:
Received 21 July 2009
Accepted 13 October 2010
Available online 19 November 2010

Keywords:
Wave propagation
Nonlinearities
Navier–Stokes equations
Power series

ABSTRACT

In this paper, a power series and Fourier series approach is used to solve the governing equations of motion in an elastic axisymmetric vessel with the assumption that the fluid is incompressible and Newtonian in a laminar flow. We obtain solutions for the wave speed and attenuation coefficient, analytically where possible, and show how these differ under a number of different conditions. Viscosity is found to reduce the wave speed from that predicted by linear wave theory and the nonlinear terms to increase the wave speed in comparison to the linear solution. For vessels with a wall stiffness in the arterial range, the reduction in the wave speed due to the viscous terms is approximately 10% and the increase due to the nonlinear terms is approximately 5%. This difference between the linear and nonlinear wave speeds was found to be largely constant irrespective of the number of terms considered in the power series for the velocity profile. The linear wave speed was found to vary weakly with stiffness, whilst the nonlinear wave speed was found to vary significantly with the stiffness, especially at low values of stiffness. The 10% variation in the wave speed due to the viscous terms was found to be constant with wall stiffness whilst the 5% variation due to the nonlinear terms was found to vary with wall stiffness. The importance of the number of terms considered in the power series is discussed showing that only a relatively small number is required in the viscous case to obtain accurate results.

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1. Introduction

Mathematical models play a valuable role in predicting flow fields and changes in vessel dimensions, as well as the influences of specific anatomical or physiological effects on the flow behaviour. This information can prove useful in understanding the processes occurring in blood vessels particularly in vascular disease conditions. Wall shear stress has been related with atherosclerosis (Caro et al., 1969, 1971) and aneurysms (Vande Geest et al., 2006). However, the underlying cause of why both vascular diseases initiate is still unclear and a matter of considerable controversy. Furthermore, there is no precise method to measure the shear stress at the vessel walls *in vivo*. There is thus much interest in the use of numerical models to understand better the flow and transport processes occurring in blood vessels.

A wide range of mathematical models that solve for the motion of blood is available. These can be one-dimensional, two-dimensional or three-dimensional. One- and two-dimensional models consider a geometrically simplified vessel, such as a straight vessel, which can give a good theoretical understanding of the underlying mechanics of any physiological effects of interest. Three-dimensional models, however, consider specific vessels which give a more precise quantitative analysis of the fluid dynamics in that individual vessel.

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doi:10.1016/j.jfluidstructs.2010.10.003

Womersley proposed a two-dimensional algorithm: a linear algorithm which was initially applied to a rigid vessel (Womersley, 1955a) assuming blood to be Newtonian, later also being applied to non-rigid vessels (Womersley, 1955b). The effects of the viscous forces on wave propagation was discussed in the latter case. It is concluded that the Womersley number, α, a dimensionless expression of the pulsatile flow frequency in relation to viscous effects, is the only parameter that governs the behaviour of oscillatory flow and that the wave speed increases with the Womersley number tending to an asymptotic value obtained by Lamb (1898). The model predicts the phase lag between the pressure gradient and the flow as well as flow reversal initiating at the walls (Hale et al., 1955). A second two-dimensional algorithm was proposed by Branson (1945). A Fourier series in time with coefficients that are functions of the radial co-ordinate was presented to solve the linear form of the equations of motion of a viscous fluid in an elastic tube. However, the equation for the conservation of mass which is presented is only valid for an inviscid fluid.

For a two-dimensional algorithm, wave propagation in non-rigid vessels has often been analysed by linearising the momentum equation. Morgan and Kiely (1954) analysed the effects of the viscous terms on wave propagation in an elastic vessel and solved for the limiting cases when $\alpha \gg 1$ and $\alpha \ll 1$. A similar approach to solve for wave propagation was considered by Atabek and Lew (1966) and Atabek (1968) for an elastic vessel and by Cox (1968) for a viscoelastic vessel.

A nonlinear two-dimensional algorithm was subsequently developed by Ling and Atabek (1972) which was applied to elastic vessels. The variation of radius with pressure was measured by *in vitro* experiment. The numerical results showed local regions of partial or complete flow reversal initiating at the walls. It was found that the linearised equations artificially increased the magnitude of the viscous forces with higher flow rates which lead to inaccurate velocity profiles compared to experimental results.

Other similar approaches where the partial differential equations of the governing equations of motion are converted to ordinary differential equations include Miekisz (1963), Rao (1983), Rashevsky (1945), Rubinow and Keller (1972). Miekisz (1963) attempted to solve the linear one-dimensional equation of motion using a Fourier series in time in an elastic tube. Rao (1983) solved for a linear two-dimensional algorithm for an elastic vessel. Steady periodic oscillations were assumed to solve for the excess pressure and thus the velocity. Rashevsky (1945) and Rubinow and Keller (1972) both obtained a solution for the variation of an elastic vessel for steady flow of a viscous fluid assuming Poiseuille flow. The latter applies this to a network of tubes.

These previous approaches have made several assumptions most notably the linearising of the equations in unsteady flow. For this purpose we propose here a novel technique which uses a nonlinear two-dimensional, axial and radial, algorithm to simulate fluid flow in a single axisymmetric vessel with no curvature. A power series over the radius and a Fourier series over time is used to represent the velocity. This is substituted into the governing equations of motion which converts the partial differential equations into a series of coupled ordinary differential equations, leading to a significant reduction in computational cost. This approach incorporates the interaction between the different power and harmonic terms, which allows for a direct quantification of the effects of the nonlinear terms. We are thus able to quantify the effects of both nonlinearities and viscous forces on wave propagation directly.

2. Theory

We consider the motion of a Newtonian incompressible fluid in an axisymmetric vessel with linear elasticity. The Navier–Stokes equation, the continuity equation and a linear elasticity equation are solved. A trial solution using a power series and a Fourier series is used to solve the equations for oscillatory flow with time period T.

2.1. Governing equations of motion

The governing equations of motion for an incompressible fluid can be obtained from the Navier–Stokes equations. In the case of an axisymmetric vessel with no curvature, using the following nondimensional variables:

$$y=\frac{r}{R},\quad z=\frac{x}{L},\quad u=\frac{U_x}{U},\quad v=\frac{U_r}{V},\quad \tau=\frac{t}{T},\quad Y=\frac{R}{R_E},$$

where x and r are the respective axial and radial co-ordinates, t is the time, R is the radius, U_x and U_r are the respective axial and radial velocities, U and V are the respective characteristic axial and radial velocities, R_E is the radius at the equilibrium state and L is a characteristic length of the vessel defined by $L=UR_E/V$, the continuity equation can be expressed as

$$Y\frac{\partial u}{\partial z}-y\frac{\partial Y}{\partial z}\frac{\partial u}{\partial y}+\frac{1}{y}\frac{\partial (yv)}{\partial y}=0. \tag{1}$$

Considering a linear pressure radius relationship (Smith et al., 2002):

$$p-p_E=G_0(Y-1), \tag{2}$$

where p is the pressure of the vessel, p_E is the pressure of the vessel at the equilibrium state and G_0 is the transmural pressure required to close the vessel or double its radius, and assuming that $\varepsilon=R_E/L\ll 1$, the momentum equation can be expressed as

$$Y\left[\mathrm{St}\left(Y\frac{\partial u}{\partial \tau}-y\frac{\partial Y}{\partial \tau}\frac{\partial u}{\partial y}\right)+v\frac{\partial u}{\partial y}+Yu\frac{\partial u}{\partial z}-y\frac{\partial Y}{\partial z}u\frac{\partial u}{\partial y}+\kappa Y\frac{\partial Y}{\partial z}\right]=\frac{1}{\varepsilon\,\mathrm{Re}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{1}{y}\frac{\partial u}{\partial y}\right), \tag{3}$$

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