Journal of the Mechanics and Physics of Solids 95 (2016) 230–238

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Some issues associated with the intermediate space in single-crystal plasticity

Morton E. Gurtin [a], B. Daya Reddy [b,*]

[a] Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, USA
[b] Department of Mathematics and Applied Mathematics, University of Cape Town, 7701 Rondebosch, South Africa

ARTICLE INFO

Article history:
Received 19 November 2015
Accepted 20 May 2016
Available online 26 May 2016

Keywords:
Finite-deformation single-crystal plasticity
Intermediate space
Lattice
Multiplicative decomposition

ABSTRACT

In this study we show that some discussions of finite-deformation single-crystal plasticity are conceptually flawed in their focus on a set referred to as the intermediate configuration. Specifically, we prove that what is usually referred to as the intermediate configuration is not a configuration but instead a vector space that we term the intermediate space. We argue that when applied to single crystals this intermediate space represents the lattice.

1. Introduction

We here consider finite-deformation plasticity based on the multiplicative decomposition $\mathbf{F} = \mathbf{F}^e\mathbf{F}^p$ of the deformation gradient $\mathbf{F}$ into elastic and plastic parts $\mathbf{F}^e$ and $\mathbf{F}^p$. Our goal is to address a conceptual flaw found in most discussions of this subject. Specifically, most discussions refer to the existence of an *intermediate configuration*. But within the context of continuum mechanics the term configuration generally connotes a *region* of Euclidean space. In contrast, we prove that the set termed the intermediate configuration is actually a *vector space* which we refer to as the *intermediate space*.

The treatment that follows in Sections 2–5 is developed within the general framework of polycrystalline plasticity; considerations specific to single crystals, such as the notion of the intermediate space as a lattice space, are the subject of Section 6.

There is no discussion of constitutive equations.

2. Kinematics of finite deformations

The space under consideration is a *three-dimensional Euclidean space* $\mathcal{E}$. The term *point* is reserved for elements of $\mathcal{E}$; the term *vector* for elements of the *associated* (three-dimensional) vector space $\mathcal{V}$. More precisely, $\mathcal{V}$ is the *translation space* for $\mathcal{E}$: given any vector $\mathbf{v} \in \mathcal{V}$ there are points $\mathbf{z}_1, \mathbf{z}_2 \in \mathcal{E}$, $\mathbf{z}_1 \neq \mathbf{z}_2$, such that $\mathbf{v} = \mathbf{z}_1 - \mathbf{z}_2$.

Consider a body B identified with the *closed* region of space it occupies in a fixed reference configuration. We refer to B as the *reference body* and denote by $\mathbf{X}$ an arbitrary *material point* of B. A *motion* of B is then a smooth mapping

* Corresponding author.
E-mail address: daya.reddy@uct.ac.za (B.D. Reddy).

http://dx.doi.org/10.1016/j.jmps.2016.05.027
0022-5096/

$$\mathbf{x} = \chi(\mathbf{X}, t) \tag{2.1}$$

with $\chi(\mathbf{X}, t)$ — for t fixed — being the *deformation* at time t, and with *deformation gradient*

$$\mathbf{F} = \nabla\chi \tag{2.2}$$

required to satisfy[1]

$$\det \mathbf{F} > 0. \tag{2.3}$$

We refer to the deformation as *homogeneous* if $\mathbf{F}$ is independent of $\mathbf{X}$.

We assume that, at each fixed t,

- $\chi(\cdot, t)$ is a one-to-one mapping of B onto $\bar{\mathrm{B}}(t)$.

Writing

$$\mathbf{X} = \chi^{-1}(\mathbf{x}, t) \tag{2.4}$$

for the corresponding inverse mapping, we can express any function of $\mathbf{X}$ as a function of $\mathbf{x}$, and vice versa; the description in terms of $\mathbf{X}$ is called the *material description*, that in terms of $\mathbf{x}$ the *spatial description*.

The closed region in space occupied by B at time t,

$$\bar{\mathrm{B}}(t) = \chi(\mathrm{B}, \mathrm{t}), \tag{2.5}$$

represents the *deformed body*. $\bar{\mathrm{B}}(t)$ is the set actually observed during the motion; the reference body B serves only to label material points: any other configuration could equally well have been used as reference. For this reason we differentiate — at least conceptually — between Euclidean space $\mathcal{E}$ as the ambient space for the reference body B and the copy of $\mathcal{E}$ that represents the space through which $\bar{\mathrm{B}}(t)$ evolves. In accord with this[2]:

(A1) the ambient space of B is referred to as the *reference space*; vectors associated with this space are referred to as *material vectors*;

(A2) the ambient space through which $\bar{\mathrm{B}}(t)$ evolves is referred to as the *observed space*; vectors associated with this space are referred to as *spatial vectors*.

It is convenient to introduce two copies,

$$\begin{aligned} \mathcal{V}_{\mathrm{mat}} &= \{\text{the space of all material vectors}\}, \\ \mathcal{V}_{\mathrm{spat}} &= \{\text{the space of all spatial vectors}\}, \end{aligned} \tag{2.6}$$

of the vector space $\mathcal{V}$ associated with $\mathcal{E}$. In addition we let

$$\mathrm{Ref} \stackrel{\mathrm{def}}{=} \{\text{reference space}\}, \quad \mathrm{Obs} \stackrel{\mathrm{def}}{=} \{\text{observed space}\}, \tag{2.7}$$

so that

$$\begin{aligned} &\mathrm{B} \subset \mathrm{Ref}, \quad \mathcal{V}_{\mathrm{mat}} \text{ is associated with Ref}, \\ &\bar{\mathrm{B}} \subset \mathrm{Obs}, \quad \mathcal{V}_{\mathrm{spat}} \text{ is associated with Obs} \end{aligned} \tag{2.8}$$

and, what is more important, each of the pairs

$$\text{Ref and Obs}, \quad \mathrm{B} \text{ and } \bar{\mathrm{B}}, \quad \mathcal{V}_{\mathrm{mat}} \text{ and } \mathcal{V}_{\mathrm{spat}} \tag{2.9}$$

is *effectively disjoint* in the sense that — conceptually — each pair has no common element.

We write $\dot{\varphi}$ for the *material time derivative* of a field φ; that is, the time derivative of φ with respect to t holding the material point $\mathbf{X}$ fixed:

$$\dot{\varphi} = \frac{\partial\varphi}{\partial t}. \tag{2.10}$$

Then

$$\mathbf{v} \stackrel{\mathrm{def}}{=} \dot{\chi} \tag{2.11}$$

represents the *velocity*. The *velocity gradient*

[1] So that $\mathbf{F}$ is invertible and orientation-preserving.
[2] Cf. Fig. 1.