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3D homogenised strength criterion for masonry: Application to drystone retaining walls

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ABSTRACT

A 3D strength criterion for masonry is constructed based on yield design theory. Yield design homogenisation provides a rigorous theoretical framework to determine the yield strength properties of a periodic medium, based on the properties of its constituent materials. First, theoretical basis of 2D homogenisation of periodic media, and more particularly its application in the framework of yield design, will be retrieved. Then, 2D principles are extended to exhibit a 3D domain of running-bond masonry. This criterion is finally used to assess the stability of a drystone retaining wall loaded by an axle load, and theoretical results are compared to experimental data. Perspectives on this work are given as a conclusion.

1. Introduction

Structural analysis of historical constructions has received a growing attention over the past decades, due to the necessary preservation of its heritage. Actually, the development of modelling proves challenging, considering the strong heterogeneity of the masonry and the diversity of its constitutive materials and patterns.

Considering the relative periodicity of their pattern, masonry can be treated as periodic composite media, and homogenisation techniques can be applied in order to derive its mechanical characteristics at macro-scale from the properties of its constituent materials. Pande et al. (1989) pioneered this technique on masonry, in order to evaluate its equivalent modulus of elasticity. This work has been extended later on by Anthoine (1995), on a 3D finite thickness pattern. Homogenisation techniques have then been extensively used to assess elastic properties (Cecchi and Sab, 2002a, 2002b; Mistler et al., 2007), or in the framework of limit analysis (Milani et al., 2006a, 2006b; Milani, 2008; Milani and Lourenco, 2010) and finite element analysis (Zucchini and Lourenço, 2002, 2004, 2009).

In 1997, de Buhan and de Felice propose a homogenisation approach for masonry developed in the framework of yield design theory. Yield design homogenisation is a rigorous theoretical approach to determine the yield strength properties of a periodic medium, based on the properties of its constituent materials. Extending on this work, the present article introduces a 3D macroscopic strength criterion for running-bond masonry, derived from the strength characteristics of blocks

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and joints.

First, theoretical basis of 2D homogenisation of periodic media, and more particularly its application in the framework of yield design, will be retrieved. This theory is then extended to the three-dimensional case, in order to exhibit a 3D macroscopic strength domain of running-bond masonry. An application of this work is finally given: the 3D strength criterion is used to assess the stability of drystone retaining wall loaded by an axle load in the framework of yield design, and theoretical results given by the model are compared to experimental data. Perspectives on this work are given as a conclusion.

2. Presentation of 2D homogenisation of periodic media principles

Homogenisation of period media consists in replacing the heterogeneous periodic medium by an equivalent homogeneous medium, which macroscopic mechanical properties are derived from the microscopic properties of the original heterogeneous medium. Introduced by Suquet (1983) in the framework of limit analysis, homogenisation technique has been extended on for yield design analysis of reinforced soils (de Buhan and Salençon, 1990), fibre composite materials (de Buhan and Taliercio, 1991), jointed rock mass (Bekaert and Maghous, 1996), and also masonry (de Buhan and de Felice, 1997).

Considering a heterogeneous periodic medium, a basic cell V can be identified as the smallest representative volume of material. For every point $\underline{x}$ of V, a microscopic strength domain $G(\underline{x})$ can be defined as the set of admissible stress fields $\underline{\underline{\sigma}}(\underline{x})$. Yield design homogenisation aims at defining the macroscopic strength domain G^{hom} of an equivalent homogeneous medium.

2.1. Static definition of G^{hom}

A static definition of the macroscopic strength domain G^{hom} can be given as the set of admissible macroscopic stress fields $\underline{\underline{\Sigma}}$:

$$G^{hom} = \left\{ \underline{\underline{\Sigma}} = \left\langle \underline{\underline{\sigma}}(\underline{x}) \right\rangle = \frac{1}{V}\int_V \underline{\underline{\sigma}}(\underline{x})\,\mathrm{d}V \right. \tag{1a}$$

$$\underline{\mathrm{div}}\ \underline{\underline{\sigma}}(\underline{x}) = \underline{0} \tag{1b}$$

$$\underline{\underline{\sigma}}(\underline{x})\cdot\underline{n}(\underline{x})\ \text{antiperiodic}\ (\underline{n}\ \text{unit normal oriented outward}) \tag{1c}$$

$$\left. \underline{\underline{\sigma}}(\underline{x}) \in G(\underline{x}) \quad \forall\ \underline{x} \in V \right\} \tag{1d}$$

2.2. Kinematic definition of G^{hom}

A kinematic definition of the macroscopic strength domain G^{hom} can be given as the set of admissible macroscopic stress fields $\underline{\underline{\Sigma}}$:

$$G^{hom} = \left\{ \underline{\underline{\Sigma}} / \underline{\underline{\Sigma}}:\underline{\underline{D}} \le \pi^{hom}(\underline{\underline{D}}) \right\} \tag{2}$$

where:

- $\underline{\underline{\Sigma}}$ is defined in Eq. (1a);
- $\underline{\underline{D}}$ is the macroscopic strain rate field given by:

$$\underline{\underline{D}} = \langle \underline{\underline{d}}(\underline{x}) \rangle = \frac{1}{2\,V}\int_V \left(\underline{\underline{\mathrm{grad}}}\ \underline{v}(\underline{x}) + {}^t\underline{\underline{\mathrm{grad}}}\ \underline{v}(\underline{x}) \right) \mathrm{d}V \tag{3}$$

- $\pi^{hom}(\underline{\underline{D}})$ is the support function of G^{hom} defined as:

$$\pi^{hom}(\underline{\underline{D}}) = \sup_{\underline{\underline{\Sigma}}} \left\{ \underline{\underline{\Sigma}}:\underline{\underline{D}} / \underline{\underline{\Sigma}} \in G^{hom} \right\} \tag{4}$$

Considering the periodicity of the medium, the virtual velocity field $\underline{v}(\underline{x})$ is given by:

$$\underline{v}(\underline{x}) = \underline{\underline{F}}\cdot\underline{x} + \underline{u}(\underline{x}) \tag{5}$$

where $\underline{\underline{F}}$ is a second order tensor and $\underline{u}(\underline{x})$ a periodic velocity field. The associated strain rate field $\underline{\underline{d}}$ can thus be written:

$$\underline{\underline{d}} = \underline{\underline{D}} + \underline{\underline{\delta}} \tag{6}$$

where $\underline{\underline{D}}$ is the symmetric part of $\underline{\underline{F}}$ and $\underline{\underline{\delta}}$ the strain rate field associated with $\underline{u}$.

de Buhan and de Felice (1997) assumed that the support function $\pi^{hom}(\underline{\underline{D}})$ of the macroscopic strength domain G^{hom} can