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Softening in random networks of non-identical beams

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ABSTRACT

Random fiber networks are assemblies of elastic elements connected in random configurations. They are used as models for a broad range of fibrous materials including biopolymer gels and synthetic nonwovens. Although the mechanics of networks made from the same type of fibers has been studied extensively, the behavior of composite systems of fibers with different properties has received less attention. In this work we numerically and theoretically study random networks of beams and springs of different mechanical properties. We observe that the overall network stiffness decreases on average as the variability of fiber stiffness increases, at constant mean fiber stiffness. Numerical results and analytical arguments show that for small variabilities in fiber stiffness the amount of network softening scales linearly with the variance of the fiber stiffness distribution. This result holds for any beam structure and is expected to apply to a broad range of materials including cellular solids.

1. Introduction

Assemblies of linear elastic beams have been used as mechanical models for materials with fibrous, cellular or periodic lattice microstructures. For example Head et al. (2003) used networks of beams to model the mechanics of biopolymer gels (also see van Dillen et al. (2008)). Beam structures have also been previously used to study cellular materials such as foams (Deshpande et al., 2001) and also in studies of the mechanics of paper (Cox, 1952). Similarly, regular lattices of beams have been used extensively in structural engineering to represent beam frameworks (Reddy, 2001).

In most of these studies the network is considered homogeneous, i.e. it is made from fibers having identical properties. However, most networks encountered in nature are composite, i.e. are made from fibers with different properties. In most biological collagen networks the fibers group in bundles of variable dimensions (Raub et al., 2007) and hence the effective network of bundles can be considered a composite network. Connective tissue gains its unique mechanical properties due to the presence of fibers such as collagen and elastin (Cowin and Doty, 2007). The cell cytoskeleton contains protein filaments such as F-actin and microtubules (Fletcher and Mullins, 2010). In papers, mixtures of fibers of different length and stiffness are used to provide enhanced strength and toughness. In all these cases the presence of the different types of fibers imparts special properties to the composite network, above and beyond those of the homogeneous networks (Broedersz et al., 2011, 2012; Buxton and Clarke, 2007; Gardel et al., 2004; Head et al., 2003; Kasza et al., 2009; Lieleg et al., 2010; Picu, 2011; van Dillen et al., 2008; Wada and Tanaka, 2009).

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To place the discussion in perspective, we briefly review the main results obtained for the relationship between network parameters and the overall stiffness of homogeneous networks (Picu, 2011). The parameters of importance in both 2D and 3D models are the network density, ρ (total fiber length per unit area or volume), the mean fiber length, l_c and the fibers axial and bending stiffness, $(EA)_f$ and $(EI)_f$. In thermal systems, the filament bending stiffness can be alternatively quantified by the fiber persistence length. For homogeneous networks, a characteristic length scale is defined as $l_b=[\frac{(EI)_f}{(EA)_f}]^{0.5}$. It has been shown that homogeneous networks with various $(EA)_f$ and $(EI)_f$ values, but with the same l_b have the same network stiffness. When ρ or/and l_b increase, the network approaches a limit in which the strain energy is predominantly stored in the axial mode of fiber deformation and the overall modulus scales linearly with $(EA)_f$ and ρ. As ρ or/and l_b decrease, the deformation is more non-affine, the strain energy is stored predominantly in the bending mode of fiber deformation and the overall modulus scales linearly with $(EI)_f$ and ρ^x. The exponent x depends on the network geometry. While Broedersz et al. (2012) and Shahsavari and Picu (2012) reported exponents of 3 and 8, respectively for 3D networks with elements aligned along the edges of a face centered cubic lattice and 2D Mikado networks respectively, here, we observe that $x = 2$ for 3D Voronoi networks. This demonstrates that the network modulus is highly sensitive to the density for systems in the non-affine, bending dominated regime.

Composite networks have been studied much less than homogeneous networks. A class of composite networks constructed by adding a small number of different fibers to a homogeneous base network was studied only recently (Bai et al., 2011; Huisman et al., 2010; Shahsavari and Picu, 2015; Wada and Tanaka, 2009). Bai et al. (2011) reported significant network stiffening from adding small fractions of stiff fibers to a non-affinely deforming base network. Stiffening was observed even when the added fibers were too sparse to form a stress-bearing network. They attributed this effect to a more affine displacement field due to the presence of the added stiff fibers. A similar type of network was studied in 3D by Huisman et al. (2010) where no stiffening effect was observed. However they reported that the presence of stiffer fibers reduces the critical strain marking the transition from the linear to the non-linear elastic regime. The problem of stiff fibers added to a non-affinely deforming base network was also studied by Shahsavari and Picu (2015). They reported that the global stiffening effect takes place in two steps. A transition is observed when the added stiff fibers percolate and form a stress-bearing network. During this process the overall stiffness increases abruptly and asymptotes to the value expected for the newly formed stiff network. A second transition takes place at smaller densities of added fibers which also leads to a substantial stiffness rise. Stiff fibers bonded to a non-affinely deforming base induce an “interphase” (i.e. a region of the base network) in which the strain energy is stored predominantly in the axial mode of fiber deformation. The second transition is associated with the percolation of these interphases.

An entirely different class of models in which a more macroscopic continuum view is taken, has been developed for fibrous structures. These are homogenized representations that lack the level of microstructural detail of the discrete models, such as those discussed above, but can be used to model much larger problem domains. Homogenized models of molecular networks have been proposed early in the development of the theory of rubber elasticity (James and Guth, 1943; Treloar, 1954). These models considered the molecules to behave as entropic springs and assumed that the motion of the cross-links is affine. Similar models based on the affine deformation assumption have been derived for athermal networks (e.g. Lee and Carnaby (1992) and Wu and Dzenis (2005)). Further developments of the theory of molecular networks partially relaxed the affine deformation restriction while considering a small number of chains as being representative for the entire network. Examples are the four chain model (Flory and Rehner, 1943; Treloar, 1954) and the eight chain model of Arruda and Boyce (1993). Improved models have been introduced that consider more broadly the non-affine deformations as well as other important factors such as structural constrains on the possible conformations of the chains (Miehe et al.,

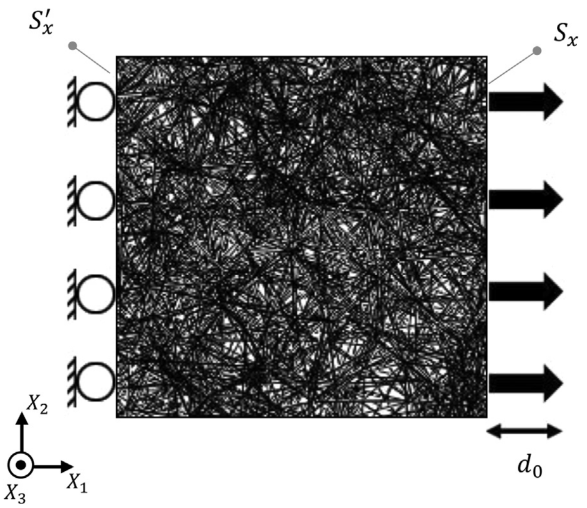

Fig. 1. Displacement controlled uniaxial stretch test of a 3D fiber network model.