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A new type of Maxwell stress in soft materials due to quantum mechanical-elasticity coupling

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ABSTRACT

All dielectrics deform when subjected to an electric field. This behavior is attributed to the so-called Maxwell stress and the origins of this phenomenon can be traced to geometric deformation nonlinearities. In particular, the deformation is large when the dielectric is elastically soft (e.g. elastomer) and negligible for most “hard” materials. In this work, we develop a theoretical framework which shows that a striking analog of the electrostatic Maxwell stress also exists in the context of quantum mechanical-elasticity coupling. The newly derived quantum-elastic Maxwell stress is found to be significant for soft nanoscale structures (such as the DNA) and underscores a fresh perspective on the mechanics and physics of polarons. We discuss potential applications of the concept for soft nano-actuators and sensors and the relevance for the interpretation of opto-electronic properties.

1. Introduction

The electrostatic Maxwell stress represents a peculiar one-way electromechanical coupling. All dielectrics deform under the action of an electric field.[1] For conventional crystalline dielectrics, e.g. Silicon, this deformation is negligible. Maxwell's stress therefore is of little significance in hard materials. Qualitatively, the mechanical strain due to the Maxwell's stress scales as $\sim\epsilon E^2/2Y$ where ϵ is the permittivity of the material, E indicates the magnitude of the electric field and Y represents the elastic stiffness of the dielectric. Evidently a softer material is more susceptible to the Maxwell stress and experiments indicate that strains of 100% and even more may be achieved in soft dielectrics like elastomers (see Fig. 1) (Pelrine et al., 2000; Keplinger et al., 2010; Li et al., 2013).

Numerous works, both classics and modern expositions, have contributed to our understanding of the Maxwell stress and related matters. For example, some of the earlier works are (Toupin, 1956; Eringen, 1963; Pao, 1978; Eringen and Maugin, 1989) and more recently, the topic has been revisited by many groups: (Dorfmann and Ogden, 2005; McMeeking and Landis, 2005; Suo et al., 2008; Zhao and Suo, 2008; Liu, 2013, 2014). The modern impetus for this topic arises due to the rather tantalizing applications of soft multifunctional materials. Unlike their hard counterparts, soft materials are usually lighter, cheaper, easily fabricated and are capable of large deformations. The potential advantages of soft materials as

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[1] The electromechanical coupling is one-way in the sense that although an electric field deforms the material, a mechanical force does not induce a polarization at the absence of an external field.

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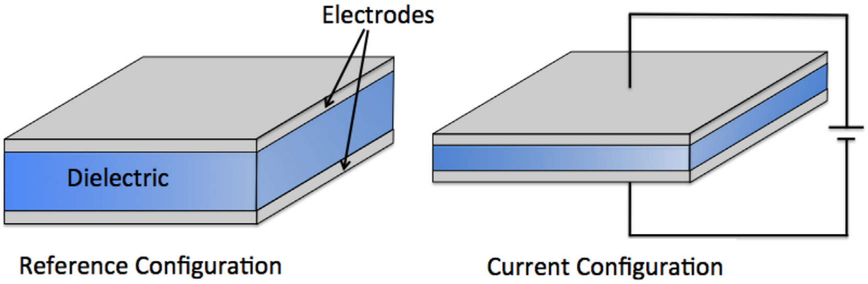

Fig. 1. The figure on the left shows the initial state (reference configuration) of the dielectric in the absence of an external electric field. The figure on the right shows the areal expansion and the reduction in the thickness due to the Maxwell stress produced by an applied electric field (current configuration).

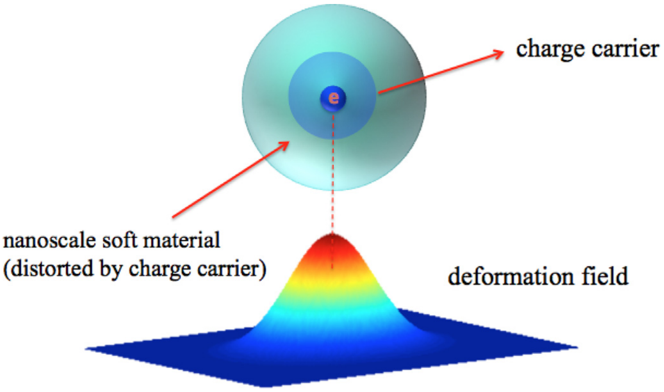

Fig. 2. A charge carrier is injected/activated into the small scale soft material and is trapped at a certain location. The charge carrier together with localized surrounding deformation forms the quasi-particle called polaron. The strain field depends on the electron–acoustic coupling strength. Once a polaron is formed, any stimuli that disturbs the quantum field (wave function) will alter the strain field thus causing additional mechanical deformation.

artificial muscles, actuators, optical fiber switches, energy harvesters, and even as medical devices, are well-documented (Fraysse et al., 2002). The many clever applications of the Maxwell stress concept in the context of soft multifunctional materials, that range from energy harvesting to soft loud speakers, are well-illustrated recently by the works of Koh et al. (2011), Zhao et al. (2007), Zhao and Suo (2010), and Li et al. (2013). In particular, Maxwell stress may be combined with the notion of electrets to design novel kinds of apparently piezoelectric materials (Kacprzyk et al., 1995; Paajanen et al., 2000; Bauer et al., 2004; Wegener and Bauer, 2005; Hillenbrand and Sessler, 2008; Deng et al., 2014a,b; Alameh et al., 2015).

In parallel to electrostatics, significant research also exists on the effect of mechanical strain on the quantum mechanical state of materials (Jiang and Singh, 1997; Johnson et al., 1998; Stier et al., 1999; Maranganti and Sharma, 2006). This topic, in particular, was strongly revitalized with the advent of the modern semiconductor technology in the early seventies. Strain is now widely used to tweak the electronic structure of quantum dots, wires and related structures—the band gap for instance. Such quantum structures, in turn, find applications in next generation lighting (Arakawa, 2002; Nakamura et al., 2002), lasers (Bhattacharya, 2000; Deppe and Huffaker, 2000) and sensors (Bhattacharya et al., 2002) among others (Grundmann et al., 1995; Tersoff et al., 1996; Bimberg et al., 1999; Williamson and Zunger, 1998; Bimberg, 1999; Bandhyopadhyay and Nalwa, 2003).

Aside from the effect of mechanical strain on the quantum state of materials, the converse effect also exists. Such an effect was first explicitly pointed out by Zhang et al. (2007) although other groups have alluded to this as well (Campbell et al., 1982; Conwell and Rakhmanova, 2000; Verissimo-Alves et al., 2001; Cristiano, 2009; Zhang et al., 2009). The central notion is that provided the structure is small enough for quantum effects to be apparent, not only does the strain impact the electronic signature of the nanostructure but also that any change in its electronic structure or quantum state may lead to a spontaneous deformation. Specifically Zhang et al. (2007, 2009) reported that the mechanical strain can be induced by solely changing the quantum field, or more generally, the electronic structure of quantum dots. Others have shown that spontaneous local deformation (distortion or tension) can be induced in some one-dimensional materials, e.g. polymer chain and carbon nanotubes (Campbell et al., 1982; Conwell and Rakhmanova, 2000; Verissimo-Alves et al., 2001; Cristiano, 2009). The physical origin of the induced deformation is the electron–acoustic phonon coupling which is also referred to as acoustic polaron.[2] For small enough nano structures, injecting a charge carrier may form a polaron—which may be

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[2] There are many other types of electron–phonon coupling such as the electron–piezoelectric polaron or electron–optical phonon couplings which are discussed in other contexts (Mahan and Hopfield, 1964; McCombe and Kaplan, 1968).