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Electro-mechanical coupling bifurcation and bulging propagation in a cylindrical dielectric elastomer tube

Tongqing Lu, Le An, Jianguo Li, Chao Yuan, T.J. Wang *

State Key Lab for Strength and Vibration of Mechanical Structures, Department of Engineering Mechanics, School of Aerospace Engineering, Xi′an Jiaotong University, Xi′an 710049, China

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This paper explores the critical and post-bulging bifurcation of a cylindrical dielectric elastomer (DE) tube undergoing finite deformation under electro-mechanical coupling loading. Explicit expressions for the critical conditions of electro-mechanical bifurcation are derived by using a simplified mathematical method. The post-bifurcation path is comprehensively investigated by specifying the material model as ideal dielectric elastomer. In the post-bifurcation analysis, we analytically establish conditions for the phase coexistence of steady propagation and analyze the physical implications. We demonstrate a global instability under force or voltage control and a localized instability under volume or charge control. Cylindrical tube experiments have been carried out under electro-mechanical coupling loading to verify the theoretical predictions. Good agreements on the critical conditions as well as the post-bifurcation path are obtained. This work characterizes the bifurcation mechanism of rubber-like materials under complex coupling loading.

1. Introduction

Various bifurcation phenomena for rubber-like materials are interesting from a theoretical point of view because they occur suddenly and deform into well-defined configurations (Gent, 2005; Cao and Hutchinson, 2012). In particular, bulging instability in inflating a rubber balloon has been intensively studied as it relates to practical applications in understanding growth of biologic issues (Humphrey, 2003), simulating diseases evolution of pressurized artery wall (Vorp, 2007), and designing balloon actuators (Fox and Goulbourne, 2008; Martinez et al., 2014). From our common experience, when a rubber balloon is inflated to a certain size it will cost less strength to blow it larger. The characteristic of transition from a stiffer balloon to a softer balloon defines a peak pressure on the pressure-stretch curve (Adkins and Rivlin, 1952). The presence of the peak pressure is due to the strong coupling of the true stress with deformation and high nonlinearity of the material. Another characteristic of inflating a rubber balloon is the stiffening effect at large stretch due to the limiting length of polymer chains. These two characteristics give the well known N-shaped pressure-stretch curve. The bulging instability is closely related to the non-monotonic behavior.

When inflating an initially flat membrane clamped at the boundary from one side, the snap-through behavior at the peak pressure is usually observed (Li et al., 2013). And the snap-back occurs in deflation. On inflating a purely spherical

* Corresponding author.
E-mail address: wangtj@mail.xjtu.edu.cn (T.J. Wang).

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membrane refined stability analysis and experiments show that the spherical configuration in inflation is unstable. The spherical membrane may bifurcate into a pear-shaped configuration through a localized thinning near one of the poles (Needleman, 1977; Haughton and Ogden, 1978; Fu and Xie, 2014). When inflating a tubular balloon, the localized bulging instability is usually observed at a critical pressure. As more fluid is pumped into the tube, the localized aneurysm develops until its radius reaches a critical value. With continuous pumping, the pressure stays unchanged and the aneurysm shifts along the tube with more and more sections bulging up at the expense of unbulged sections. The localized bulging for a tubular balloon has been experimentally studied in Kyriakides and Chang (1991), Pamplona et al. (2006). For analytical studies, some focus on bifurcation analysis and stability analysis (Shield, 1972; Haughton and Ogden, 1979; Chen, 1997; Chen and Haughton, 2003), and some focus on the steady propagation after the bifurcation without solving inhomogeneous deformation fields (Yin 1977; Chater and Hutchinson, 1984). The effects of axial load and boundary conditions are also considered in Alexander (1971), Kyriakides and Chang (1990). A full analysis tracing the whole deformation process can be described with a simpler mathematical method by transforming the differential governing equations to algebraic equations (Fu et al., 2008; Fu and Xie, 2010).

All the previous investigations on bulging instability only deal with purely mechanical loading. We find that charging a dielectric elastomer (DE) shares the common characteristics with inflating a rubber balloon. Due to their outstanding properties of large deformation, light weight, silent operation, low cost and fast response, dielectric elastomers are promising for applications as actuators, generators, and sensors (Pelrine et al., 2000; Carpi et al., 2010; Brochu and Pei, 2010). A membrane of a dielectric elastomer sandwiched between two compliant electrodes undergoes large deformation subjected to a voltage across the thickness, e.g. voltage-induced areal strain greater than 1000% (Keplinger et al., 2012; Godaba et al., 2014) and the updated deformation record 2200% (An et al., 2015). The voltage induced deformation is due to Columbic force and thus the electric field determines the intensity. Like the coupling between the true stress and stretch in inflating a rubber balloon, the electric field couples strongly with deformation. The Columbic force squeezes the membrane and the decreased thickness amplifies the electric field. The positive feedback competes with the elasticity of membrane, inducing the electrical instability (Stark and Garton, 1955; Zhao and Suo, 2007). Similarly, the electrical instability defines a peak voltage on the voltage-stretch curve. The curve eventually becomes N-shaped due to strain stiffening. This analogy on the N-shaped behavior provides much convenience to understand the voltage-induced behavior based on the well-known force-induced behavior.

For the case of the electro-mechanical coupling loading, most attention has been paid on hard materials, such as piezoceramics and ferro-electrets (Lynch, 1996; Qu and Yu, 2011). Few studies address the electro-mechanical coupling instability and bifurcation of soft materials undergoing finite deformation, such as soft piezo materials, soft electrets and electrostrictive polymers (Bauer et al., 2004). The electro-mechanical coupling instability of dielectric elastomers has been developed due to promising applications in electro-mechanical transducers (Zhao and Wang, 2014). We usually eliminate the instability to prevent failure of electrical breakdown (Koh et al., 2011; Lu et al., 2012). But sometimes the instability can be harnessed to achieve large actuation (Keplinger et al., 2012). However, to the authors′ knowledge, few studies address the electro-mechanical coupling bifurcation and post-bifurcation of soft materials. The main difficulties are the complicated inhomogeneous finite deformation and the highly nonlinear electro-mechanical coupling.

In this work we analyze the electro-mechanical bifurcation and post-bifurcation of cylindrical DE tube undergoing finite deformation. We focus on the localized bulging instability and the subsequent steady propagation. In particular, in constructing the general electro-mechanical bifurcation condition we make use of a simpler mathematical method to avoid solving full inhomogeneous fields. By specifying the material model as ideal dielectric elastomer, we analyze two kinds of instability under different loading conditions. Finally we conduct experiments on a cylindrical DE tube under the internal pressure, axial load, and voltage across the thickness. Localized bulging and steady propagation are clearly observed. Good

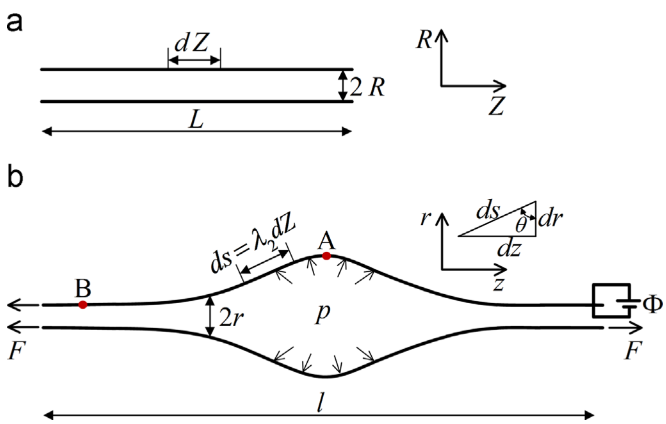

Fig. 1. Schematics of a cylindrical rubber tube subjected to an internal pressure, an axial force and a voltage across the thickness. The deformation is axisymmetric and inhomogeneous.