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Broadband elastic metamaterial with single negativity by mimicking lattice systems

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ABSTRACT

The narrow bandwidth is a significant limitation of elastic metamaterials for practical engineering applications. In this paper, a broadband elastic metamaterial with single negativity (negative mass density or Young's modulus) is proposed by mimicking lattice systems. It has two stop bands and the bandwidth of the second one is infinite theoretically. The effect of the relevant parameters on band gaps is discussed. A continuum model is proposed and the selection of materials is discussed in detail. It shows that continuum metamaterials can be described accurately by using the lattice model, and the second stopband can be ultra-broad but not infinite. This discrepancy is investigated and a method is provided to calculate the upper limit of the second stopband for a continuum metamaterial. As a verification, the proposed metamaterial is used for wave mitigation over broadband frequency ranges. Moreover, the present method is extended to design 2D anisotropic elastic metamaterials, and a device to control the direction of elastic wave transmission is proposed as an example.

1. Introduction

Driven by the rapid development of electromagnetic/optic metamaterials with negative permittivity and permeability (Pendry et al., 1996, 1999; Smith et al., 2000; Shalaev, 2007), acoustic/elastic metamaterials have gained much attention in the last decade (Liu et al., 2000; Li and Chan, 2004; Fang et al., 2006; Ding et al., 2007; Huang et al., 2009a; Milton and Willis, 2007; Lee et al., 2010; Lai et al., 2011; Liu et al., 2011; Yang et al., 2013). Analogous to negative permittivity and permeability in electromagnetic metamaterials, the central focus of acoustic/elastic metamaterials is on the negative effective modulus and mass density, which are not found in natural materials. It should be noted that the aforementioned negative modulus and mass density are dynamic effective parameters, which are different from the static negative stiffness discussed by Lakes et al. (2001) and Lakes and Drugan (2002). Structures with a static negative stiffness are usually unstable unless they are coated within a positive-stiffness matrix (Drugan, 2007). The first metamaterial with negative mass was investigated and fabricated by Liu et al. (2000) based on localized dipolar resonances. The acoustic metamaterial with negative bulk modulus was firstly designed by Fang et al. (2006) using an array of Helmholtz resonators. Ding et al. (2007) demonstrated that acoustic metamaterials with double negativity can be achieved by combining structures having negative modulus and mass density independently. Subsequently, structures with double negativity were proposed based on

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different mechanisms (Lee et al., 2010; Lai et al., 2011; Liu et al., 2011). In order to illustrate the physical meaning and mechanisms of negative effective mass and modulus of acoustic/elastic metamaterials, lattice systems with masses and springs were used to construct simplified models (Huang et al., 2009a; Milton and Willis, 2007; Huang and Sun, 2011a). Meanwhile, many novel applications were proposed in accordance with the unusual properties of acoustic/elastic metamaterials, such as invisibility cloak (Zhang et al., 2011), superlens (Ambati et al., 2007; Li et al., 2009; Zhou and Hu, 2011), negative refraction (Liu et al., 2011; Wu et al., 2011) and vibration attenuation devices (Yang et al., 2010; Yao et al., 2010; Tan et al., 2012; Zhu et al., 2014; Huang and Sun, 2009b).

However, for most acoustic/elastic metamaterials mentioned above, the negative effective mass and modulus only exhibit over a narrow frequency region. Thus the design of structures with broadband negative parameters is an important issue in the field of acoustic/elastic metamaterials. Thin membrane-type structures (Lee et al., 2010; Yang et al., 2010, 2013) were proposed as broadband metamaterials to control acoustic waves. Yao et al. (2010) showed that the effective mass can be negative under a specific frequency by fixing internal resonators. Recently, multiple local resonators (Tan et al., 2012; Zhu et al., 2014) and graded resonators (Baravelli and Ruzzene, 2013) have also been used to extend the width of bandgaps to some extent. It is worth mentioning that waves cannot propagate in lattice systems, such as monatomic and diatomic chains, above a certain frequency, which corresponds to an ultra-broad forbidden band. Therefore, ultra-broadband negative modulus or mass density may be obtained for structures with the feature of lattice systems.

In light of this, a model of broadband elastic metamaterial with single negativity is investigated here by mimicking lattice systems. The effective parameters obtained by using a two-step homogenization method fit the dispersion relations perfectly. It shows that this discrete model has two stopbands, and the bandwidth of the second one is infinite in theory. Then the way to design continuum metamaterials is discussed in detail. The second forbidden band of the continuum model, although not infinite, is quite broad. As a demonstration, the proposed metamaterial is used for wave mitigation over broadband ranges. Finally, a device of controlling the direction of elastic wave transmission is presented as an application of 2D anisotropic elastic metamaterials.

2. Lattice model

2.1. Negative effective mass and modulus

Consider an infinite mass-spring system as shown in Fig. 1(a), which was proposed by Huang et al. (2009a) as a one-dimensional model of elastic metamaterials with negative mass density. Masses m_1 are connected periodically at a spacing of L by springs with stiffness K. There is a mass m_2 connected by the spring k in each m_1. The dispersion equation of this system can be derived as

$$m_1 m_2 \omega^4 - [(m_1 + m_2)k + 2m_2 K(1 - \cos(qL))]\omega^2 + 2Kk(1 - \cos(qL)) = 0 \quad (1)$$

where ω is angular frequency and q is wave number. As an example, the dispersion curve is plotted in Fig. 2(a) in the case of $\theta = m_2/m_1 = 2$ and $\delta = k/K = 1$. There are two bandgaps of this structure in the range of $0.885 < \omega/\omega_0 < 1.732$ and $3.196 < \omega/\omega_0 < +\infty$ respectively, where $\omega_0 = \sqrt{k/m_2}$ is the local resonance frequency of m_2.

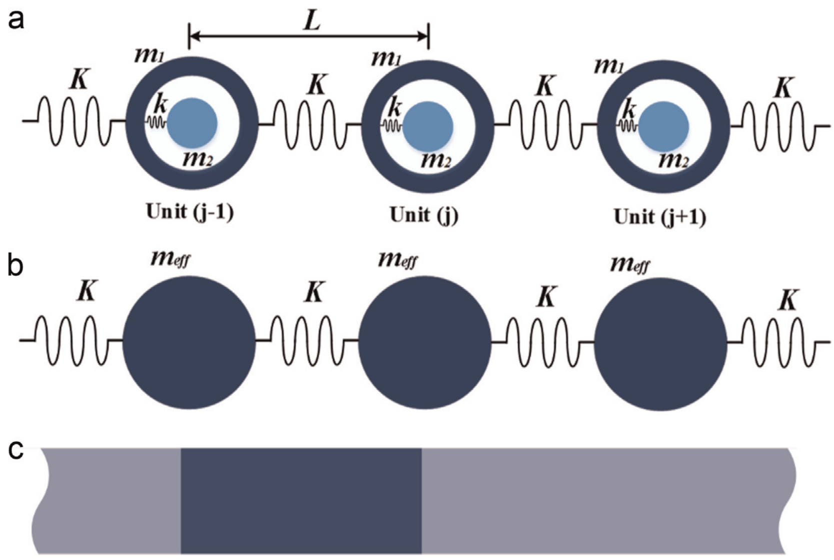

Fig. 1. Infinite mass-in-mass lattice system and its effective models.