



Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

About the influence of hydrostatic pressure on the yielding and flow of metallic polycrystals[☆]

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ARTICLE INFO

Article history:

Received 25 July 2013

Received in revised form

12 February 2014

Accepted 14 February 2014

Available online 3 March 2014

Keywords:

Crystal plasticity

Homogenization

Yield surface

Flow rule

ABSTRACT

We consider an aggregate of single crystals featuring pressure dependent yielding and deviatoric plastic response, and inquire about the overall response of the aggregate. Since this is one of the prototype examples of what is usually called non-associated plastic flow, the investigation is focused upon discerning the essential features of the resulting model at macro-scale: yield surface and flow rule. It is shown that a flow rule relates the macro-direction of plastic deformation, an additional macro-variable (which vanishes in the absence of pressure effects) and the exterior normal of the macro yield surface. Furthermore, if the constituent crystals feature uncoupled volumetric–deviatoric elastic response, then the plastic dilatancy is zero, the equivalent stress function is pressure-independent, and the flow rule reduces to the classical normality rule in the subspace of deviatoric stress tensors.

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1. Introduction

Plastic deformation of metals has long been considered to be pressure independent meaning that, up to certain stress levels, yielding at a continuum material particle is not affected by the hydrostatic component of the stress state at that particle. According to Hill (1950), the origins of this fundamental axiom of classical plasticity may be traced back to the early experimental work of Polanyi and Schmid (1923) and Bridgman (1946), who showed it to be true at least “to a first approximation”. The second fundamental axiom of classical (phenomenological) plasticity, known as the normality rule, states that the direction of the rate of plastic deformation is along the exterior normal to the yield surface issued at the current stress state. In conjunction with the normality rule, the pressure independence axiom implies as primal consequence that the volumetric part of the rate of plastic deformation is always zero.

On the other hand, later experimental work conducted by Spitzig and Richmond (e.g., Richmond, 1982; Spitzig and Richmond, 1984 and references therein) has shown that yielding is affected by the hydrostatic component of stress, though the magnitude of the effect is consistent with a theory incorporating second order effects, while the plastic deformation is essentially purely deviatoric (in the absence of micro-voids).² These findings seem to contradict the axiom of normality,

[☆] In memory of Dr. Owen Richmond.

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since, to quote from [Spitzig and Richmond \(1984\)](#), “there is no need for the pressure dependency of yielding to be associated with irreversible plastic dilatancy”. As later remarked in [Bulatov et al. \(1999\)](#), this is equivalent to stating that the normality rule holds in isobaric sections through the yield surface.

The pressure effect upon the yielding of metals is just an instance of a more general feature: the Schmid law, stating that a slip system of a constituent crystal becomes (potentially) active only when its corresponding resolved shear stress reaches a critical value, may not be an entirely accurate description of what happens in reality, for other stress components could also influence slip activity ([Asaro and Rice, 1977](#); [Dao et al., 1996](#); [Vitek, 2005](#); [Bassani and Racherla, 2011](#)). The general analysis in [Soare \(2014\)](#) shows that the main consequence at macro-scale of assuming non-Schmid effects at micro-scale (constituent level) is that the direction of the macro-rate of plastic deformation is deviated from the exterior normal to the macro yield surface, but that, at the same time, is still related to the latter via a generalized flow rule involving an additional macro-variable, this being representative of the non-Schmid effects affecting yielding at micro-scale.

When particular non-Schmid effects are considered, the description of the resulting macro-model can, in principle, be performed at a higher resolution. From this perspective, the analysis in [Soare \(2014\)](#) is reconsidered here in the context of pressure effects. The relationship between the macro-rate of plastic deformation and the overall yield surface of a polycrystal aggregate is rederived via a direct approach, with new and specific details.

1.1. Pressure-dependent yielding and strength-differential: a discussion of some of Spitzig and Richmond's experiments with hydrostatic pressure

A pressure independent yielding criterion is usually written in the form $f(\boldsymbol{\sigma}) := g(\boldsymbol{\sigma}') - \sigma_h = 0$ where $\boldsymbol{\sigma}' := \boldsymbol{\sigma} - [\text{tr}(\boldsymbol{\sigma})/3]\mathbf{I}$ denotes the deviatoric part of the stress tensor, g denotes the so-called equivalent stress, and σ_h denotes a measure of the strength of the material, often estimated with simple uniaxial traction tests (determining the hardening curves). At the outset, no symmetry is assumed for the yield function f . In particular, the elastic domain defined by f may not have a center of symmetry with respect to the zero stress state, that is, $g(\boldsymbol{\sigma}) \neq g(-\boldsymbol{\sigma})$. It is assumed that g is the first order positive homogeneous.

Motivated by the results of their experiments, [Spitzig et al. \(1975, 1976\)](#) introduce pressure dependence in the above yielding criterion in the form $g(\boldsymbol{\sigma}') + q \text{tr}(\boldsymbol{\sigma}) = \sigma_h$, with $q \geq 0$ a material parameter which is, similar to σ_h , dependent on the strain history, e.g., the equivalent plastic strain. Let the stress state associated with a uniaxial test with superimposed hydrostatic pressure be $\boldsymbol{\sigma}_u := \tau \mathbf{u} \otimes \mathbf{u} + p \mathbf{I}$, with deviator $\boldsymbol{\sigma}'_u = \tau \mathbf{A}$, where $\mathbf{A} := \mathbf{u} \otimes \mathbf{u} - (1/3)\mathbf{I}$. Here $\tau > 0$ if tensile, and $\tau < 0$ if compressive. Also, $p < 0$ for hydrostatic compression; \mathbf{u} is a unit direction along the loading axis. Assuming for the moment that the yield function f is normalized so that $g(\mathbf{A}) = 1$, yielding under $\boldsymbol{\sigma}_u$ is characterized by $\tau = \sigma_h(1 - 3qp/\sigma_h)/(1 + q)$. [Spitzig and Richmond \(1984\)](#) define $\alpha := q/\sigma_h$ and $\sigma_0 := \sigma_h/(1 + q)$, so that the previous relationship is rewritten in the form $\tau = \sigma_0(1 - 3\alpha p)$, and regard it as the main expression of the influence of hydrostatic pressure upon yielding. However, this form is specific to a uniaxial load and to recover the whole generality the pressure dependent yield criterion is rewritten now in the form

$$g(\boldsymbol{\sigma}') = \sigma_h[1 - \alpha \text{tr}(\boldsymbol{\sigma})] \quad (1)$$

One of the main conclusions of Spitzig and Richmond's experiments with pressure can be concentrated in the statement: α is a *constant* material parameter. Their measurements show it to be quite small, e.g., $\alpha \approx 56/\text{TPa}$ for aluminum, and $\alpha \approx 13\text{--}23/\text{TPa}$ for iron and steels. This is in agreement with the general consensus that the effect of the hydrostatic component of the stress state on yielding is small and hence it can be neglected within an appropriate order of approximation.

Since pressure dependence leads to a certain asymmetry of the yield surface, with respect to origin of the stress space, estimating the magnitude of this asymmetry can lead to an estimation of the order of the above approximation. This is helpful in contexts where accurate modeling of the yield surface is required, or where the pressure dependence of the yield criterion is explicitly accounted for. Then considering the tension–compression asymmetry of the yield surface, define $g^T := g(\mathbf{A})$ and $g^C := g(-\mathbf{A})$. Substituting in (1) the stress state $\boldsymbol{\sigma}_u$ leads to the following formulas for the uniaxial stress strain curves in tension and compression, respectively: $\tau^T = \sigma_h(1 + 3\alpha|p|)/(g^T + \alpha\sigma_h)$ and $\tau^C = (\sigma_h(1 + 3\alpha|p|))/(g^C - \alpha\sigma_h)$. A measure of the strength differential can be defined as follows:

$$SD := 2(\tau^C - \tau^T)/(\tau^C + \tau^T) = 2(g^T - g^C + 2\alpha\sigma_h)/(g^T + g^C) \quad (2)$$

In this form it is possible to discern between what may be called intrinsic strength differential, represented here by the g^T and g^C ad hoc material parameters, and the strength differential caused by the hydrostatic component of the stress state. In particular, if the intrinsic asymmetry is absent, then, say, $g^T = g^C = 1$, and the magnitude of the strength differential is calculated by ([Spitzig and Richmond, 1984](#))

$$SD = 2\alpha\sigma_h \quad (3)$$

In this (intrinsically symmetric) case, the strength differential is always positive: dependence of the yield criterion on the hydrostatic component implies that the yield strength in compression is higher than in tension. To cite two examples from [Spitzig and Richmond \(1984\)](#), for the 4330 steel the initial yield stress is $\sigma_h(\epsilon = 0) = 1480 \text{ MPa}$ and $\alpha = 20/\text{TPa}$, leading to the prediction $SD = 0.058$, which compares well with the value of 6.0% measured from experiments. Conversely, this prediction implies that the intrinsic strength differential along the particular tested axis is negligible, at least in the initial stages of

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