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An analytical model for the translaminar fracture toughness of fibre composites with stochastic quasi-fractal fracture surfaces

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ABSTRACT

The translaminar fracture toughness of fibre-reinforced composites is a size-dependent property which governs the damage tolerance and failure of these materials. This paper presents the development, implementation and validation of an original analytical model to predict the tensile translaminar (fibre-dominated) toughness of composite plies and bundles, as well as the associated size effect. The model considers, as energy dissipation mechanisms, debonding and pull-out of bundles from quasi-fractal fracture surfaces; the corresponding lengths are stochastic variables predicted by the model, based on the respective bundle strength distributions and fracture mechanics. Parametric studies show that composites are toughened by stronger fibres with large strength variability, and intermediate values of interfacial toughness and friction. Predictions are validated against four different composite ply systems tested in the literature, proving the models ability to capture not only size effects, but also the influence of different fibres and resins.

1. Introduction

The translaminar fracture toughness ($\mathcal{G}$) of unidirectional (UD) fibre reinforced polymers (FRPs) is the energy required to fracture the material perpendicular to the fibre direction (per unit nominal area). This property governs the damage tolerance of structures with load-aligned fibres, as well as the strength of components with geometric discontinuities; it is also dramatically affected by size effects (Laffan et al., 2012), which raises a challenge for the simulation of damage tolerant structures. This paper presents a model for the translaminar tensile toughness of FRPs, based on fibre and interfacial properties and assuming the formation of stochastic quasi-fractal fracture surfaces.

The translaminar toughening mechanisms of FRPs have been extensively investigated (Kim and Mai, 1991), and methods to measure the corresponding fracture toughness have been developed (Laffan et al., 2012). Composites are orders of magnitude tougher than their constituents, due to the formation of intricate 3D fracture surfaces with large interfacial debonds and pulled-out fibres and bundles (Figs. 1 and 2).

Laffan et al. (2010) recently reported size effects on the translaminar toughness of FRPs, by testing cross-ply Compact Tension (CT) specimens with 0.125 or 0.250 mm thick 0° layers. The measured translaminar toughness of the thicker layers was nearly twice the value for the thinner ones, reportedly due to much larger pull-out features (Fig. 1). Subsequent finite

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Nomenclature

Uppercase variables

A	cross-sectional area
C	perimeter
CoV	coefficient of variation
E	elastic modulus
F	cumulative distribution function (CDF)
$\mathcal{F}$	extreme-value CDF
$\mathcal{G}$	(translaminar) fracture toughness
L	length (stochastic)
$\mathcal{L}$	extreme-value length (stochastic)
S	survival probability, complementary CDF
$\mathcal{S}$	extreme-value complementary CDF
V	volume fraction
W	energy dissipated in fracture
X	longitudinal tensile strength (stochastic)

Lowercase roman variables

a	debonding distance
c	coordination number
g	translaminar toughness components
i	bundle level
k	stress concentrations factor
l	length
m	Weibull shape parameter
n	number of fibres
s	interfibre spacing
t	thickness

Lowercase greek variables

α	aspect ratio
ϕ	diameter
κ	stress concentrations parameter (Eq. (14))
λ	stress field slope
σ	longitudinal stress
τ	shear (yield) stress
ψ	toughness parameter (Eq. (14))

Superscripts

0	single-fibre composite
$\mathtt{f}$	fibre
$[i]$	bundle level
∞	remote

Subscripts

0	Weibull scale parameter
$\mathtt{deb}$	debonding
$\mathtt{H}$	hexagonal arrangement
II	mode-II delamination
m	mean value
μ	frictional
$\mathtt{po}$	pull-out
$\mathtt{Q}$	quadrangular arrangement
$\mathtt{r}$	reference length
$\mathtt{SL}$	interface, shear-lag
$\mathtt{U}$	uniform stress state

element simulations of open-hole specimens (Chen et al., 2013) proved that incorporating such dependence in numerical models is crucial to replicate experimental results.

Additionally, Pimenta et al. (2010) observed that the fracture surface of recycled-fibre bundles is hierarchical and statistically self-similar (see Fig. 2a, with individual fibres pulled-out from the surface of small bundles, which are themselves pulled-out from larger bundles). Further analysis confirmed that these features are also characteristic of virgin UD composites (Fig. 2b).

Most authors (e.g. Gao et al., 1988; Kelly, 1970; Kim and Mai, 1991; Wells and Beaumont, 1985b) agree that interfacial debonding and pull-out (hereby indicated by the subscripts $\mathtt{deb}$ and $\mathtt{po}$) are the main toughening mechanisms of UD

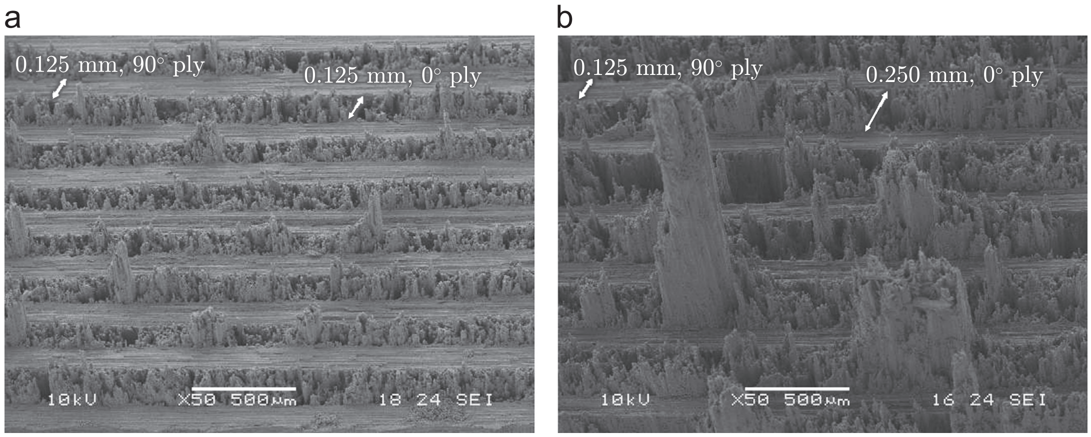

Fig. 1. Size effects on the translaminar fracture of UD carbon–epoxy plies (after Laffan et al., 2010). (a) Fracture surface with 0.125 mm thick 0° layers and $\mathcal{G} = 65\ \mathrm{kJ/m^2}$. (b) Fracture surface with 0.250 mm thick 0° layers and $\mathcal{G} = 132\ \mathrm{kJ/m^2}$.