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# A homogenization approach for nonwoven materials based on fiber undulations and reorientation

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ABSTRACT

This paper presents a new micromechanical based approach for the modeling of the highly anisotropic and non-linear stiffening response of fibrous materials with random network microstructure at finite strains. The first key aspect of the proposed approach arises from the experimentally justified need to model the elastic microscopic response of the constituent fibers, which are one-dimensional elements, as linear elastic. This linear elastic behavior is modified in the lower strain regime to account for the inherent fiber undulations and the associated fiber unfolding phenomena. Another key aspect is the reorientation of these fibers which is identified as one primary mechanism for the overall macroscopic stiffening. The one-dimensional elements are statistically distributed as unit vectors in a non-uniform manner over an affine referential network space of orientations represented by a unit circle in the two-dimensional case of interest here. A physically motivated reorientation of these unit vectors is achieved by a bijective map which asymptotically aligns them with the maximum loading direction in the referential orientation space. A rate-independent evolution law for this map is sought by a physically motivated assumption to maintain the overall elastic framework of the proposed formulation. A closed form solution to the new evolution law is also presented which allows faster computation of updating orientations without resorting to numerical integration or storing history variables. The unit vectors upon reorientation in the referential orientation space are then mapped to the spatial orientation space by the macrodeformation gradient to compute the macroscopic Kirchhoff stress and the associated spatial elasticity modulus. Reorientation of these unit vectors results in the evolution of the associated probability density function which is also computed in closed form depending on the initial probability density. However, it is shown that for a bijective reorientation map, the homogenization of micro-variables over the referential orientation space is independent of the evolving probability density function. Homogeneous deformation tests are performed to highlight the elastic framework of the proposed formulation. A direct comparison of the numerical results with the experimental results from the literature is made which demonstrates the predictive capabilities of the proposed formulation.



## 1. Introduction

Materials made from synthetic advanced fibers have a very high strength-to-weight ratio getting them rapidly absorbed in numerous industrial applications with a demanding environment. *Aramid* fibers like Nomex, Kevlar, Twaron, Technora are

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one such class of synthetic advanced fibers which are made from polymerization of aromatic polyamides. Another such class of fibers is produced from *ultra-high-molecular-weight polyethylene* e.g. Dyneema, Spectra which have recently gained more popularity in industry due to their even higher strength-to-weight ratio. Applications like industrial slingers, environmental friendly fishery nets, medical equipments, nanofilters, textile reinforced concretes, ballistic apparels, military and space equipments are just a few among innumerable to be listed here. It becomes indispensable to understand the mechanics of materials made from such advanced fibers like their deformation response or failure behavior under extreme loadings. A special focus is on materials with random fiber network (RFN) microstructure made from these advanced fibers which are produced by bonding or interlocking networks of randomly laid fibers through a mechanical needle-punching process. These materials are commonly referred to as *nonwoven* materials.

Earliest experimental investigations (Backer and Petterson, 1960; Cusick et al., 1963; Hearle and Stevenson, 1963; Hearle and Newton, 1968) of nonwoven materials described these materials as highly anisotropic with a non-linear load–displacement response exhibiting inhomogeneous deformations during monotonic loading. Thirlwell and Treloar (1965) studied the lateral contraction of such materials and found that they do not obey Saint-Venant′s principle, i.e. stresses are non-uniformly distributed even at distances longer than the order of lateral dimension. Further experiments (Zhang et al., 1998; Chocron et al., 2002; Rawal, 2006; Hou, 2010; Ridruejo et al., 2011; Afshari et al., 2012) on these materials provided more knowledge about their deformation mechanism under different loading conditions such as in-plane tensile loading or out-of-plane impact loading. Experiments by Pourdehyhimi et al. (1999), Ghassemieh et al. (2002, 2002b) or Miao and Glassey (2004) studied effects of fiber orientation distribution where techniques borrowed from image processing were used for the analysis. Recent experiments by Jearanaisilawong (2008) and Chocron et al. (2008) on needle-punched nonwovens made from *Dyneema* fibers shed more light on their complex deformation mechanism which is observed to be governed by fiber unfolding and stretching, inter-fiber friction, volume compaction due to change of volume fraction of fibers, reorientation of fibers, disentanglements due to fiber slipping from junctions or fiber breakage. Measuring the effect of these quantities to successfully predict the material response is a key challenge for any newly proposed numerical model.

One of the first continuum based modeling approaches can be found in Backer and Petterson (1960) and Petterson and Backer (1963) which was based on the orthotropic theory of elasticity. Such an approach lacks the capability to capture the non-linear load–displacement curves due to the absence of any microscopic information fed into macroscopic continuum model. By using the laminate theory of composites, Bais-Singh and Goswami (1995) proposed a 'layer theory' where the microstructural information about the fiber orientation distribution is directly incorporated into a unit cell, which is made from piling up differently oriented layers of mono-directional fibers. Various improvements like adding transverse and shear stiffness to a layer were suggested in Bais-Singh et al. (1998) and Bais-Singh and Goswami (1998), however, the microstructural aspects like fiber unfolding, slipping or bending, fiber reorientation or stiffening during deformation due to fixed fibers within each layer could not be captured. Cox (1952) developed a relation for the in-plane stiffness of fibrous materials in terms of Fourier coefficients of the distribution function only valid for small strains. Based on this, Petterson (1958) formulated a 'fiber web theory' which also performs poorly for large strains and lacks any information about fiber undulations, slip or reorientation inside the unit cell. Hearle and Stevenson (1963, 1964) incorporated the effects of initial fiber undulations, Kothari and Patel (2001) included creep response of individual fibers and Narter et al. (1999) extended the 'fiber web theory' to three dimensions resulting in an improved overall predicting capability of the model. In another class of method (Treloar and Riding, 1963; Hearle and Newton, 1967, 1968), the stress–strain relations of a fabric were derived from energy relations where the total energy stored in the fabric is computed from the fiber properties and the applied external strain. The compressional behavior of such materials was first studied by van Wyk (1946) by taking fiber bending and the number of fiber-to-fiber contacts into account, which was followed by Carnaby and Pan (1989), Lee et al. (1992) and Komori and Itoh (1994) who made certain modifications to account for initial anisotropy, friction, slippage and viscous effects.

The microstructure of nonwoven materials can be seen as a randomly cross-linked network of fibers, which has some analogies to the microstructure of elastomers, where a randomly cross-linked network of polymer chains is found. This led Jearanaisilawong (2008) to adopt continuum based *network models* commonly used to model rubber elasticity. Early network models are the *three-chain model* by James and Guth (1943) and Wang and Guth (1952), where the macroscopic deformation is affinely related to three mutually orthogonal chains in a cubic space of orientations, and the *four-chain model* by Flory and Rehner (1943) and Treloar (1946), where the macro–micro deformation is linked non-affinely via four chains in a tetrahedral space of orientations. More recent developments of the network models can be found in the *eight-chain model* by Arruda and Boyce (1993), a *full network model* by Wu and van der Giessen (1993), the *non-affine microsphere model* by Miehe et al. (2004), the *maximal advance path constraint* model by Tkachuk and Linder (2012) or the *modified eight-chain model* by Purohit et al. (2011). These models are based on approximating the microscopic polymer chain behavior with the Langevin statistics to correctly predict the macroscopic response at large deformations. To model the nonwoven material behavior, the *eight-chain model* by Arruda and Boyce (1993) was modified in Jearanaisilawong (2008) where the initial anisotropy of the material was introduced in terms of a fabric tensor which was derived from the experimental fiber orientation distribution. Thirteen material parameters were used in the numerical simulations which match very well with the experimental results in the range from small to mid strains. However, at larger strains, the predictive capability of the numerical model deviates from the experimental results. Nevertheless, modeling the behavior of a single fiber with Langevin statistics which has asymptotic property at the limiting stretch contrary to its physically observed linear elastic behavior is questionable. In addition, the length scale of the constituent fiber is much larger as compared to a polymer chain so that the contribution of the entropic free energy can be neglected.