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Effective moduli of ellipsoidal particle reinforced piezoelectric composites with imperfect interfaces

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The effective properties of piezoelectric composites with ellipsoidal particles embedded imperfectly in the matrix are investigated. The dilute approximation method, the Mori–Tanaka method, the self-consistent method, and the differential scheme are all modified to incorporate the bonding imperfection to predict the effective elastic, dielectric, and piezoelectric moduli of the composite. The corresponding formulae are rigorously derived with the help of the modified piezoelectric Eshelby tensor. Numerical examples are considered to illustrate the effect of imperfect interfaces on the effective properties of piezoelectric composites. It is found that good agreement with the existing experiments can be achieved by properly selecting the interface parameters. This clarifies the importance of the inclusion of imperfect interfaces in the modeling. The particular size-dependent characteristic due to the interface imperfection is also investigated numerically.

1. Introduction

Composite materials with different combinations of matrix and reinforcing elements have found wide applications in modern technologies and industries. They are microscopically heterogeneous, but can be homogenized at a macroscopic level to facilitate the analyses for engineering applications.

Usually, the macroscopic properties of composites can be estimated by two different classes of approaches. The first class is based on variational principles and stems from the pioneering work of Hashin and Shtrikman (1962a, 1962b, 1963). Within this framework, the lower and upper bounds for the effective properties of multiphase particulate composites can be estimated mathematically, see for instances Walpole (1966a, 1966b, 1969, 1972), Beran and Molyneux (1966), and Willis (1977).

The second class is generally known as the effective medium approach, emanating from Eshelby's classic study (Eshelby, 1957). Analytical micromechanical methods of the second class are very popular since they allow us to predict multi-axial properties and responses of heterogeneous materials in principle. The popular models include the dilute method, the Mori–Tanaka method (Mori and Tanaka, 1973; Taya and Mura, 1981; Benveniste, 1987; Chen et al., 1992), the differential

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scheme (McLaughlin, 1977; Hashin, 1988), the self-consistent method (Hill, 1965; Budiansky, 1965; Budiansky and O′connell, 1976), and the generalized self-consistent method (Christensen and Lo, 1979).

We note that, in the study of Eshelby (1957), the inclusion is assumed to be perfectly bonded to the matrix. In practice, however, the interfaces between the matrix and the inclusions in composite materials may not be perfect, with the imperfection introduced or caused possibly in the process of material fabrication and product manufacture. So, in addition to the properties of individual constitutive phases, the interfaces play a key role in controlling the overall properties of a composite. In this case, the assumption of perfect bonding may not be adequate, and to accurately predict the overall properties of the composite, it is necessary to take account of the effect of imperfect interfaces between the inclusions and the matrix.

The elastic case considering imperfect interface has been addressed by many investigators (Achenbach and Zhu, 1989; Hashin, 1990, 1991a, 1991b; Gao, 1995). The adopted imperfect interface models in their works assumed that the stresses are continuous across the matrix-inhomogeneity interface, while depending on the type of the imperfection some components of the displacements are discontinuous. The continuous stresses are proportional to the displacement jumps, and hence these spring-like imperfect models are in an algebraic form. The effective elastic moduli of composite materials in the presence of imperfect interfaces between the inclusions and the matrix were recently investigated by Yanase and Ju (2012) by using the modified elastic Eshelby solution for an algebraic interface model. Similar algebraic interface models also exist in the regime of thermal conduction analysis, and they have been adopted in the study of effective thermal conductivities of heterogeneous media (Duan and Karihaloo, 2007a).

On the other hand, the surface/interface effects may become significant in nano-composites (i.e. when the reinforcing components are at a nano-scale), and should be taken into consideration. Duan et al. (2005a, 2005b, 2006) and Duan and Karihaloo (2007b) have conducted a series of investigations on the Eshelby problem of nano-sized inclusions and demonstrated the intrinsic size-dependence of effective properties of nano-composites. In their analyses, the generalized Young–Laplace equations of surface elasticity, which are essentially partial differential equations, have been adopted to model the surfaces/interfaces. A quite complete survey on the subject has been given by Duan et al. (2009), where the reader could find more interesting discussions on such differential interface models and their effects as well as a complete list of references.

The recent two decades have witnessed the huge expansion of piezoelectric materials in applications of smart structures and systems, due to the inherent coupling between the electric and mechanical fields in these advanced functional materials. Though, the low value of the hydrostatic strain coefficient d_h obviously limits their applicability. This drawback has driven the development of piezoelectric composites which exhibit higher values of d_h than the piezoelectric ceramic alone (Newnham et al., 1978). This is attributed to the complicated electromechanical interactions occurring in the combination of the piezoelectric and matrix phases. To further the development of piezoelectric composites for electromechanical transducers and emerging smart material applications, it is crucial to develop reliable theories and methods to predict the effective properties of piezoelectric composites (Deeg, 1980). By assuming perfect bonding conditions, Dunn and Taya (1993) successfully extended the well-established elastic analysis to the piezoelectric case. Jiang et al. (1999) presented a unified model for piezocomposites based on Budiansky′s energy-equivalence framework. Few works, however, have been concerned with developing methods to predict the effective properties of piezoelectric composites with imperfect interfaces. In Appendix A of this work, the modified piezoelectric Eshelby tensor is presented, where an algebraic interface model has been adopted to take account of the effect of imperfect interfaces.

The objective of this study is to understand the effect of imperfect interfaces on the effective properties of a piezoelectric composite, and to provide a way for interpreting the existing experimental results properly. Several well-known micromechanical methods are extended here to the piezoelectric case, in which both the electromechanical coupling and interfacial bonding imperfection are taken into consideration. The formulations are derived based on the modified piezoelectric Eshelby tensor for ellipsoidal particles, and are the extension of the purely elastic case (Yanase and Ju, 2012). The developed methods are then used to show numerically the effect of imperfect interfaces on the effective electroelastic moduli of piezoelectric composites containing ellipsoidal inhomogeneities. In particular, comparison with reported experiments indicates the necessity of building complex factors (e.g. the imperfect interface) into the theoretical modeling of practical advanced composites.

2. Basic equations in Barnett–Lothe notation

Consider the most general case of piezoelectricity, which takes full consideration of material anisotropy and coupling between the electric and elastic fields. The basic equations can be expressed in a fixed rectangular coordinate system x_i $(i=1,2,3)$ as

$$\sigma_{ij}=c^E_{ijmn}\varepsilon_{mn}+e_{nij}(-E_n),\quad D_i=e_{imn}\varepsilon_{mn}-\kappa^{\varepsilon}_{in}(-E_n), \tag{1}$$

$$\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}),\quad E_i=-\phi_{,i}, \tag{2}$$

$$\sigma_{ij,j}=0,\quad D_{i,i}=0, \tag{3}$$