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Study on the mechanisms and quantitative law of mode I supersonic crack propagation

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ABSTRACT

Continuum mechanics predicts that the propagation speed of non-equilibrium information in solids is limited by the longitudinal wave speed, so is crack propagation. However, solids are essentially discrete systems. In this paper, via theoretical analysis and numerical simulations, it is demonstrated in a straightforward way that non-equilibrium disturbance (e.g. force, displacement, energy, and so on) can propagate at a supersonic speed in discrete systems, although the magnitude of the disturbance attenuates very quickly. In dynamic fracture, a cascade of atomic-bond breaking events provides an amplification mechanism to counterbalance the attenuation of the disturbance. Therefore, supersonic crack propagation can be realized in a domino way. Another key factor for supersonic crack propagation is to ensure sufficient energy flowing into the crack tip. Since most energy can only be transferred at a speed limited by the longitudinal wave speed, the conditions for the occurrence of supersonic crack propagation are not easily met in most situations, unless there is high pre-stored energy along the crack path or continuous energy supply from the loading concomitantly moving with the crack tip. A quantitative relation between supersonic crack propagation speed and material properties and parameters is given, which implies that knowing all the classical macroscopic quantities is not enough in determining the supersonic crack propagation speed, and the microstructure does play a role. Moreover, it is interesting to note that fracture toughness affects the crack propagation speed in the subsonic regime, but not in the supersonic regime, because the deformation/stress is uniform in front of a supersonic crack where strength criterion dominates.

1. Introduction

Rapidly propagating crack has been studied for many years, but lots of phenomena have still not been understood thoroughly. Especially, what is the upper limit of crack propagation speed attracts much attention recently, and there is a lack of consensus on the answer to this question. In the framework of continuum mechanics, dynamic fracture mechanics predicts that the energy release rate for mode I crack becomes negative if the crack propagates faster than the Rayleigh wave speed, implying that the Rayleigh wave speed is the upper limit of mode I crack speed (Broberg, 1999; Freund, 1998; Slepyan, 2002). Previous numerical simulations (Abraham et al., 1994, 1997; Buehler et al., 2004b; Rountree, 2002; Swadener et al., 2002) and experimental work (Cramer et al., 2000; Fineberg et al., 1991; Hauch et al., 1999; Hauch and

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Marder, 1998; Marder and Gross, 1995; Ravi-Chandar, 1998) showed that crack does not propagate beyond the Rayleigh wave speed. For a mode II crack, continuum mechanics also predicts a negative energy release rate if the crack propagates faster than the Rayleigh wave speed, except in the range of intersonic propagation (between the shear and longitudinal wave speeds), which implies that a mode II crack can propagate at intersonic speed. Burridge (1973) attributed the mechanism to the positive peak of shear stress ahead of the tip of the mode II crack tip traveling at the Rayleigh wave speed. Andrews (1976) found that when the speed of crack propagation approaches to the Rayleigh wave speed, a microcrack with the velocity beyond the shear wave speed will be induced ahead of the mother crack tip. Taking advance of molecular dynamics (MD) method, Abraham and Gao et al. (Abraham and Gao, 2000; Gao, 2001; Abraham, 2001) simulated the mode II crack propagation at the velocity exceeding shear wave speed and found a daughter crack formed ahead of the mother crack tip. Needleman (1999) obtained the similar results using the cohesive surface in finite element simulations. The experimental observation of Rosakis et al. (1999; 2002) provided the first direct evidence that a mode II crack can travel at intersonic speed.

However, it is also noted that continuum mechanics fails to predict the following numerical and experiments results on dynamic fracture. Gao (1997) and Abraham (1996, 1997) observed in their MD simulations that a mode I crack propagates at super-Rayleigh speed, and a mode II crack propagates at supersonic speed. They attributed the phenomenon to the local hyperelastic effects of solids (Buehler et al., 2003, 2004a; Buehler and Gao, 2006; Guo et al., 2003). Also using MD method to investigate dynamic fracture, Guozden et al. (2005; 2010) concluded that both mode I and mode III cracks can propagate at supersonic speed. Petersan et al. (2004) conducted dynamic fracture experiments on popping rubber, and found that the crack propagation speed of a mode I crack is faster than the shear wave speed. Slepyan (1972, 1981, 2001a, 2001b) pioneered the theoretical study on the possibility of supersonic crack propagation in lattice system and presented the corresponding analytical solutions. Adopting similar theoretical analyses, Mishuris et al. (2008, 2009) studied the bridge crack propagation in lattice system and found that the supersonic crack propagation speed was predicted in the analytical results.

The contradiction between the predictions from continuum mechanics and those from experimental and MD results on the upper limit of crack propagation speed has brought a lot of confusion. On the one hand, according to continuum mechanics theory, any information of non-equilibrium disturbance cannot travel faster than the longitudinal wave speed. If so, how can a crack propagate faster than this upper limit speed? On the other hand, the MD simulations and experiments indeed demonstrated that the Rayleigh wave speed, or even the longitudinal wave speed, can be exceeded in mode I crack propagation. These contradictive predictions inspire us to suspect that the essential discreteness of solids might play an important role in rapid dynamic fracture, which is neglected or smeared out in continuum mechanics. We would like to emphasize that all solids consist of discrete atoms, and continuum mechanics is only an idealized model of solids based on continuity assumption. In most cases, continuum mechanics can give good predictions and have been widely used. However, the fracture of solids involves atomic bond breaking at the crack tip, and continuum mechanics might yield incorrect predictions in certain extreme cases.

This paper is aimed to explore the underlying mechanisms of supersonic crack propagation, and study the quantitative dependence of supersonic crack propagation speed on both material properties and microstructure parameters. The structure of this paper is as follows. In Section 2, we show several numerical cases in which a mode I crack propagates at a supersonic speed. In Section 3, by investigating the difference between the dynamic behaviors of one dimensional discrete and continuous solid system, we demonstrate that the information of non-equilibrium disturbance can propagate at a supersonic speed, which is the key factor validating the possibility of supersonic propagation of a mode I crack. In Section 4, we investigate several typical loading conditions, and reveal suitable energy supply mechanisms for maintaining supersonic crack propagation. In Section 5, the effects of microstructure and other parameters on supersonic crack propagation are discussed. Conclusions are summarized in Section 6.

2. Numerical examples of supersonic mode I crack propagation

To systematically study the dynamic fracture behaviors, a two-dimensional strip specimen with triangle lattice shown in Fig. 1a is simulated. This is an idealized model for a solid crystal, in which concentrated mass m is only deployed at nodes (or atoms) and the connecting bonds are represented by massless linear springs with original length l_0 and spring constant k. The simulation system consists of 120 rows, and each row has 450 nodes. At first, vertical tensile displacement loading is applied to the top and bottom boundaries of the non-cracked specimen, and the system is under uniform deformation with stretched bond length l_a. A crack is then suddenly introduced by removing the left part of bonds crossing the horizontal middle plane while keeping the displacement loading fixed as shown in Fig. 1a. In order to simulate a straight propagating crack, it is assumed that the rest of the bonds crossing this middle plane (colored by red) have a critical breaking length l_c, while the other bonds never break. The dynamical finite element method (FEM) (ABAQUS, 2005) is used to simulate the subsequent dynamic fracture behaviors. The focus of this numerical study is to investigate whether a mode I crack can propagate at a supersonic speed.

We first determine the longitudinal wave speed of this two dimensional network by simulation and theoretical analysis. Fig. 2a shows three sequential snapshots of the non-cracked specimen after a pulse dilatational loading on the left part of the specimen, from which the longitudinal wave speed can be computed as $c_l = 1.061 l_0 \sqrt{k/m}$. The following theoretical analysis further validates this computed value. According to continuum mechanics, the elastic constants of a