Journal of the Mechanics and Physics of Solids 59 (2011) 18–42

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Instabilities in multilayered soft dielectrics

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ARTICLE INFO

Article history:
Received 3 March 2010
Received in revised form
16 July 2010
Accepted 2 October 2010

Keywords:
Electroactive polymers
Bifurcations
Localization of deformation
Composite materials
Finite elasticity

ABSTRACT

Experimental observations clearly show that the performance of dielectric elastomeric-based devices can be considerably improved using composite materials. A critical issue in the development of composite dielectric materials toward applications is the prediction of their failure mechanisms due to the applied electromechanical loads. In this paper we investigate analytically the influence of electromechanical finite deformations on the stability of multilayered soft dielectrics under plane-strain conditions. Four different criteria are considered: (i) loss of positive definiteness of the tangent electroelastic constitutive operator, (ii) existence of diffuse modes of bifurcation (*microscopic* modes), (iii) loss of strong ellipticity of the homogenized continuum (localized or *macroscopic* modes), and (iv) electric breakdown. While the formulation is developed for generic isotropic hyperelastic dielectrics, results are presented for the special class of ideal dielectrics incorporating a neo-Hookean elastic response. The effect of material properties and loading conditions is investigated, providing a detailed picture of the different possible failure modes.

1. Introduction

The application of a voltage through electrodes to soft dielectric elastomers deforms them substantially, giving us the opportunity to use this principle to design a new class of actuators. Discovery and development of these materials were first reported in the works of Pelrine and coworkers (Pelrine et al., 1998, 2000). Immediately, they attracted significant interest because of tremendous potential in areas as robotics, aerospace and biomedical engineering. Currently, dielectric elastomers are widely employed to manufacture devices as reliable electrically driven actuators, manipulators and energy harvesters (see Bar-Cohen, 2001; Carpi et al., 2008a; Carpi and Smela, 2009, and references cited therein).

A significant challenge in the development of devices based on dielectric elastomers is that they often require the application of extremely high voltages as a result of the material low dielectric constant. This represents a clear limitation in their development toward further applications, but both experimental (Zhang et al., 2002; Huang et al., 2004; Carpi et al., 2008b) and analytical (deBotton et al., 2007) investigations showed that composite materials can provide a solution to this critical issue. When stiff and high-permittivity particles are included in a soft elastomeric matrix, the overall dielectric constant of the material increases considerably, while its deformability may be only marginally affected. deBotton et al. (2007) showed that the use of biphasic laminate dielectric composites can improve the actuation strain up to 50% or, on the other hand, can provide the same actuation with a sensible decrease of the applied voltage. However, to

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0022-5096/$ - see front matter
doi:10.1016/j.jmps.2010.10.001

design a new class of optimized devices based on dielectric composites, further investigations are necessary focusing both on their failure under the applied loads and on the effect of inclusions volume fraction, geometry and material properties on their performance. In this work we concentrate on the latter issue, investigating in a systematic way instabilities that develop in finitely deformed multilayered dielectrics.

The optimization of the performance of dielectric elastomer actuators is a challenging task due to their multiple failure modes. For single-phase actuators, electromechanical instabilities (unstable thinning of the actuator, local buckling induced by coexistent states, electric breakdown) have been explored both analytically (Zhao and Suo, 2007; Zhao et al., 2007; Moscardo et al., 2008; Liu et al., 2009) and experimentally (Plante and Dubowsky, 2006), providing also design guidelines to prevent potential failures under operating conditions. Moreover, Dorfmann and Ogden (2010) recently investigated surface instabilities for an electroelastic half space. In composite systems instabilities are even more critical and a larger family of potential failure modes must be considered.

In this scenario modeling represents a fundamental tool. Motivated by the development of applications, the nonlinear theory of soft dielectrics, first proposed by Toupin (1956), has been recently reviewed and further developed. In particular, we refer to the work of McMeeking and Landis (2005), Dorfmann and Ogden (2005) and Suo et al. (2008), where both the concepts of Maxwell and total stresses and the formulation of constitutive equations for conservative materials have been discussed. In addition, based on this theory a variational formulation has been built and discretized using the finite element method (Vu et al., 2007).

In this paper the stability of multilayered hyperelastic dielectric elastomers deforming at large strains is systematically investigated. Four instability criteria for composites are introduced,[1] namely

(i) *loss of positive definiteness* of the tangent electroelastic constitutive operator;
(ii) existence of diffuse modes of bifurcation (*microscopic* modes);
(iii) loss of strong ellipticity of single phases and of the homogenized continuum (localized modes or, in the latter case, *macroscopic* modes);
(iv) *electric breakdown*.

The first three criteria follow from the theory of bifurcation and stability for nonlinear elastic solids developed by Hill (1957) and Biot (1965) and applied subsequently to investigate loss of uniqueness of given loading paths in boundary-value problems for both homogeneous solids (Hill and Hutchinson, 1975; Ogden, 1984; Needleman and Ortiz, 1991; Triantafyllidis and Lehner, 1993) and composite materials (Triantafyllidis and Maker, 1985; Geymonat et al., 1993; Bigoni and Gei, 2001; Triantafyllidis et al., 2006; Michel et al., 2007). Electric breakdown is instead specific for dielectric materials that are characterized by a limit maximum value for the intensity of the electric field, beyond which electric discharges may take place.

The four instability criteria are then specialized to rank-one *layered composites* finitely deformed under plane-strain conditions. A detailed analysis of instabilities is reported for a multilayer with two phases made of ideal dielectrics incorporating a neo-Hookean elastic response (Dorfmann and Ogden, 2005). Interestingly, the results clearly show that depending on the heterogeneity contrast between the phases and on the loading conditions different failure modes may occur.

2. Theory of elastic dielectrics

2.1. Basic notation

In this section we summarize the equations governing the nonlinear electrostatic deformation (electrodynamical effects are excluded) of heterogeneous dielectrics following the formulation previously introduced by McMeeking and Landis (2005), Dorfmann and Ogden (2005), deBotton et al. (2007) and Suo et al. (2008).

Let us consider an isolated system consisting of a multi-phase electroelastic body and the complemental surrounding space (Fig. 1) and indicate by $\mathcal{B}^0$ and $\mathcal{B}^0_{\rm sur} = \mathbb{R}^3 \backslash \mathcal{B}^0$ the undeformed stress-free configuration of the body and the surrounding space, respectively. We identify with $\partial\mathcal{B}^0$ the boundary separating $\mathcal{B}^0$ from the surrounding, while $\partial\mathcal{B}^0_{\rm int}$ denotes the set of all the internal interfaces between heterogeneities in $\mathcal{B}^0$. The application of both mechanical loadings and electric fields deforms quasi statically the body from $\mathcal{B}^0$ to the current configuration $\mathcal{B}$ and interfaces $\partial\mathcal{B}^0$ and $\partial\mathcal{B}^0_{\rm int}$ to $\partial\mathcal{B}$ and $\partial\mathcal{B}_{\rm int}$, respectively. Such deformation is described by the function $\boldsymbol{\chi}$ that maps a reference point $\mathbf{x}^0$ in $\mathcal{B}^0$ to its deformed position $\mathbf{x} = \boldsymbol{\chi}(\mathbf{x}^0)$ in $\mathcal{B}$. The associated deformation gradient will be denoted by $\mathbf{F} = \partial\boldsymbol{\chi}/\partial\mathbf{x}^0$, while J identifies its determinant, $J = \det\mathbf{F}$. If the surrounding space does not consists of vacuum, the deformation $\boldsymbol{\chi}$ can be extended to $\mathcal{B}^0_{\rm sur}$, yielding $\mathcal{B}_{\rm sur} = \mathbb{R}^3 \backslash \mathcal{B} = \boldsymbol{\chi}(\mathcal{B}^0_{\rm sur})$.

At this point it is important to note that when electric and/or magnetic interactions are considered, the solution of a boundary-value problem requires the integration of the governing equations over the entire system (i.e. the body and the

[1] It is important to note that throughout the paper we refer generically to (i)–(iv) as 'instability' criteria. More precisely, (i)–(iii) detect a possible bifurcation point along the fundamental deformation path, while (iv) corresponds to a failure threshold for the solid.