Mechanics of Materials 102 (2016) 15–25

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

Research paper

# Effects of crack surface electrostatic tractions on the fracture behaviour of magnetoelectric composite materials

Aibing Zhang[a,*], Baolin Wang[b]

[a] Piezoelectric Device Laboratory, School of Mechanical Engineering & Mechanics, Ningbo University, Ningbo, 315211, PR China
[b] Institute for Infrastructure Engineering, Western Sydney University, Penrith, NSW 2751, Australia

ARTICLE INFO

*Article history:*
Received 4 June 2015
Revised 13 June 2016
Available online 10 August 2016

*Keywords:*
Magnetoelectric
Electrostatic tractions
Boundary condtions
Crack

ABSTRACT

The fracture mechanics problem of a Griffith crack embedded in a two-dimensional magnetoelectric composite material subjected to coupling mechanical, electric and magnetic loads at infinity is investigated. The crack surfaces electrostatic tractions are taken into account and the crack is assumed to be electrically and magnetically semi-permeable. Explicit and closed form solutions of electric displacement and magnetic induction inside the crack, stress, electric displacement and magnetic induction intensity factors are derived based on the extended Stroh formalism and continuous distribution of generalized dislocation approach. Numerical computations are also carried out to illustrate the influence of electrostatic tractions on the crack tip field when the crack interior is filled with different dielectric medium. It is found that the electrostatic tractions on the crack surfaces have the tendency to close the crack thus retard the crack propagation, and the traditional traction-free crack model always overestimates the effect of applied magnetoelectric loads on the crack tip filed intensity factors. Crack surfaces electrostatic tractions cannot be neglected for large applied electric or magnetic load to mechanical load ratio, and small dielectric constant and magnetic permeability of the crack interior.



## 1. Introduction

Magnetoelectric composite materials made of piezoelectric and piezomagnetic phase have been found extensively applications in modern industries such as sensors, actuators, transducers, hydrophones, electric packaging and acoustic/ultrasonic devices due to coupling mechanical, electric and magnetic effect (Alshits et al., 1992; Huang and Kuo, 1997; Nan, 1994). However, because of their brittleness and low strength, cracks and flaws in such composites inevitably occur either during manufacturing process or when subjected to mechanical, electric and magnetic loads. Thus, fracture mechanics problems in magnetoelectric composite materials have been received tremendous interest. Green's functions for an infinite two-dimensional magnetoelectroelastic medium containing an elliptical cavity or a crack were derived based on the extended Stroh formalism and complex function method by Liu et al. (2001). Crack initiation behavior in magnetoelectric composite materials under in-plane deformation was studied by Song and Sih (2003). The interaction problem of multiple arbitrarily oriented and distributed cracks in magnetoelectroelastic materials was considered by Tian and Gabbert (2004). Green's functions for the various defects embedded in an infinite magnetoelectroelastic material under thermal loading were obtained with the framework of in-plane magnetoelectroelastic interactions by Qin (2005). García-Sánchez et al. (2007) analysed that the behavior of cracked linear magnetoelectroelastic materials by using the dual boundary element method approach. Recently, to eliminate the singularity behavior on the crack tips, a pre-fracture zone on electrically impermeable and magnetically permeable interface crack model between two dissimilar magnetoelectroelastic materials was developed by Ma et al. (2013). The solutions for electrically and magnetically impermeable crack branching out of the crack plane in a magnetoelectric composite materials under thermo-electro-magneto-mechanical loads were presented by Zhang and Wang (2014a). Hu et al. (2015) investigated interface crack problem by extending the concept of Dugdale crack model and Yoffe model to a moving crack between two dissimilar magnetoelectroelastic materials. For more contributions in the literature on the fracture mechanics of magnetoelectric composite materials, see Wang and Shen (2002), Gao et al. (2003), Sih and Song (2003), Qin (2004), Soh and Liu (2005), Ootao and Tanigawa (2005), Tupholme (2009), Rojas-Díaz et al. (2012), Hu et al. (2014), Sladek et al. (2015), Bhargava et al. (2015), Rogowski (2015) and Ma et al. (2015).

In above-mentioned works, four different boundary conditions on the crack surfaces, there are electrically impermeable and mag-

* Corresponding author.
*E-mail address:* zhangaibing@nbu.edu.cn, zabtt@163.com (A. Zhang).

http://dx.doi.org/10.1016/j.mechmat.2016.08.007

netically permeable, electrically permeable and magnetically impermeable, electrically and magnetically impermeable and electrically and magnetically permeable are assumed to investigate the fracture mechanics problem in magnetoelectric composites. Wang and Mai (2007) discussed the applicability of the crack-face electromagnetic boundary conditions, and pointed out that the electric and magnetic permeabilities of air or vacuum inside the crack cannot be ignored. Taking electromagnetic properties of crack interior into consideration and using magnetoelectric boundary conditions at the crack face related to crack opening displacement, the fracture mechanics problem of magnetoeletroelastic solid was solved by Zhong and Li (2007). To date, the crack surfaces stress boundary conditions of most magnetoelectric composites crack problems are still considered to be free, which is identical with the typical assumption for the fracture mechanics analysis of purely elastic materials. Generally, an opened crack filled with air or vacuum represents an obstacle against an electric field perpendicular to the crack plane, since the dielectric constant of air or vacuum is at least three orders of magnitude smaller than that of magnetoelectric composites. Electric field that passes the crack gap causes a normal electric potential jump across the crack and induces electrostatic stresses that pull the crack surfaces together (McMeeking and Landis, 2005; Neumeister et al., 2013). The electrostatic tractions acting upon crack faces should be introduced in piezoelectric medium fracture evaluation together with the electrically semi-permeable crack model, especially when the mechanical loads is not too large and the electric displacement loads is not too small (Ricoeur and Kuna, 2009). The axisymmetric problem of a penny-shaped crack embedded in an infinite three-dimensional piezoelectric body is considered by Li et al. (2011). The effect of electrostatic tractions on the fracture behavior of a piezoelectric material under mechanical and/or electric loading is analyzed by Xie et al. (2014) and Zhang and Wang (2014b).

However, in contrast to the piezoelectric materials, very few papers can be found on the influence of crack surfaces electrostatic tractions on magnetoelectric composite materials fracture mechanics problem. On the other hand, due to the pronounced piezoelectric, piezomagnetic and magnetoelectric properties, the electric displacement on the crack surfaces does not vanish and has a non-negligible magnitude even if only a mechanical (or magnetic induction) loads is applied at infinity to a magnetoelectric composite material with a crack filled with air/vacuum. Therefore, the primary objective of this paper is to investigate effect of crack surfaces electrostatic tractions on the fracture behaviour of magnetoelectric composites. The plan of the paper is as follows. The extended Stroh formalism is outlined in Section 2. In Section 3 the explicit solutions of stress, electric displacement and magnetic induction intensity factors are derived for magnetoelectric composite materials problem. Some numerical results are presented in Section 4, and concluding remarks are made in Section 5.

## 2. The Stroh formalism

Consider a linear magnetoelectroelstic material in which all fields are assumed to depend only on the in-plane coordinates $x_1$ and $x_2$. The basic equations (constitutive, divergence and gradient) of the linear magnetoelectric composite solids without any body force, electric charge and electric current are (Liu et al., 2001):

$$\begin{aligned}\sigma_{ij} &= c_{ijkl}u_{k,l} - e_{lij}\phi_{,l} - h_{lij}\varphi_{,l}\\ D_i &= e_{ikl}u_{k,l} + \kappa_{il}\phi_{,l} + \alpha_{il}\varphi_{,l}\\ B_i &= h_{ikl}u_{k,l} + \alpha_{il}\phi_{,l} + \mu_{il}\varphi_{,l}\end{aligned} \tag{1}$$

$$\sigma_{ij,i} = 0, D_{i,i} = 0, B_{i,i} = 0 \tag{2}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), E_i = -\phi_{,i}, H_i = -\varphi_{,i} \tag{3}$$

In Eqs. (1)–(3), a comma denotes partial differentiation and repeated indices mean summation. $\sigma_{ij}$, $\varepsilon_{ij}$, $D_i$, $E_i$, $B_i$ and $H_i$ are elastic stress, strain, electric displacement, electric field, magnetic induction and magnetic field, respectively. $u_i$, $\phi$ and $\varphi$ are elastic displacement, electric potential and magnetic potential, respectively. $c_{ijkl}$, $e_{ikl}$, $h_{ikl}$, $\alpha_{ij}$, $\kappa_{ij}$ and $\mu_{ij}$ are the elastic moduli, piezoelectric coefficients, peizoemagnetic coefficients, magnetoelectric coefficients, dielectric constants and magnetic permeability coefficients, respectively.

For two-dimensional deformations, the general solution to the Eq. (1) can be written as

$$\mathbf{u} = \mathbf{Af}(z) + \overline{\mathbf{Af}(z)} \tag{4}$$

where $\mathbf{u} = [u_1, u_2, u_3, \phi, \varphi]^{\mathrm{T}}$, $\mathbf{f}(z) = diag[f(z_1), f(z_2), f(z_3), f(z_4), f(z_5)]$ is arbitrary analytic function, $z_i = x_1 + p_i x_2$, the superscript "T" and the overbars stand for transposition of a matrix and complex conjugation, respectively. $p_i$ and $\mathbf{A}$ are constants determined by

$$\left[\mathbf{Q} + p_i(\mathbf{R} + \mathbf{R}^{\mathbf{T}}) + p_i{}^2\mathbf{T}\right]\mathrm{A}_i = 0 \tag{5}$$

in which $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{T}$ are defined by

$$\mathbf{Q} = \begin{bmatrix} c_{1ik1} & e_{1j1} & h_{1j1} \\ e_{1k1}^T & -\kappa_{11} & -\alpha_{11} \\ h_{1k1}^T & -\alpha_{11} & -\mu_{11} \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} c_{1ik2} & e_{2j1} & h_{2j1} \\ e_{1k2}^T & -\kappa_{12} & -\alpha_{12} \\ h_{1k2}^T & -\alpha_{12} & -\mu_{12} \end{bmatrix},$$

$$\mathbf{T} = \begin{bmatrix} c_{2ik1} & e_{2j2} & h_{2j2} \\ e_{2k2}^T & -\kappa_{22} & -\alpha_{22} \\ h_{2k2}^T & -\alpha_{22} & -\mu_{22} \end{bmatrix}$$

The stress, electric displacement and magnetic induction obtained by inserting Eqs. (3) and (4) into Eq. (1) can be expressed in terms of the generalized stress function vector $\mathbf{\Phi}$ as

$$\begin{aligned}\mathbf{t}_1 &= [\sigma_{11}, \sigma_{12}, \sigma_{13}, D_1, B_1]^{\mathrm{T}} = -\mathbf{\Phi}_{,2}\\ \mathbf{t}_2 &= [\sigma_{21}, \sigma_{22}, \sigma_{23}, D_2, B_2]^{\mathrm{T}} = \mathbf{\Phi}_{,1}\end{aligned} \tag{6}$$

where

$$\mathbf{\Phi} = \mathbf{Bf}(z) + \overline{\mathbf{Bf}(z)} \tag{7}$$

$$\begin{aligned}\mathbf{B} &= \mathbf{R}^{\mathrm{T}}\mathbf{A} + \mathbf{TAP}\\ \mathbf{P} &= diag[p_1, p_2, p_3, p_4, p_5]\end{aligned} \tag{8}$$

## 3. Fracture mechanics solution of magnetoeletric composite materials by considering crack surface electrostatic tractions

Consider a generalized 2D magnetoelectric composite material problem of a single crack of length $2a$and filled with air or vacuum, as shown in Fig. 1. The opening displacement of the crack is always very small, the electric displacement and magnetic induction through crack should not be zero. Therefore, it is more reasonable to consider the electric and magnetic fields inside the crack and the electric and magnetic potential jumps across the crack surfaces simultaneously, that is magnetoelectric semi-permeable crack faces boundary condition. The remote loading system is composed of stresses $\sigma_{21}^{\infty}$, $\sigma_{22}^{\infty}$ and $\sigma_{23}^{\infty}$, electric displacement $D_2^{\infty}$ and magnetic induction $B_2^{\infty}$, i.e.,

$$\mathbf{t}_2^{\infty} = [\sigma_{21}^{\infty}, \sigma_{22}^{\infty}, \sigma_{23}^{\infty}, D_2^{\infty}, B_2^{\infty}]^{\mathrm{T}} \tag{9}$$

The faces of a non-conducting crack in a dielectric material are interfaces between a solid and air, vacuum or any other insulation matter. If either an electric loading is imposed or the material exhibits physical properties coupling magnetical/mechanical