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Research paper

Assessment of the effective parameters of dual porosity deformable media

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ABSTRACT

This paper deals with the estimation of the effective parameters of double porosity isotropic deformable media paying particular attention to the effective viscoelastic effect. In this type of media two fluid-saturated pore networks of very different permeabilities are interconnected. This situation leads to a transient and inhomogeneous pressure diffusion state in the micropores that gives rise to effective *viscoelastic* properties at the macroscopic scale. The study is conducted in three steps. First, a methodology is developed that allows to reformulate the results previously obtained by homogenization within a self-consistent framework that enables the analytical estimation of the effective parameters. Then, the developed approach is applied to two basic double porosity morphologies, i.e. large pores embedded in a microporous domain and a microporous domain with a small amount of highly permeable fractures. In both cases the effective bulk and shear moduli, compressibility, and pressure-deformation coupling factor are determined. A parametric study of the analytical results follows. This leads to identify, for each effective parameter, the expected magnitude of the viscoelastic effect and the corresponding characteristic frequency according to (i) the type of morphology, (ii) the porosity and characteristic dimension of the Representative Elementary Volume, and (iii) the values of the physical parameters of the solid and fluid constituents.

Nomenclature

Microporous domain

- The porosity of the microporous domain is denoted as ϕ.
- $\boldsymbol{\alpha}$ is the symmetric and positive Biot coupling tensor. For an isotropic material this becomes $\boldsymbol{\alpha} = \alpha \mathbf{I}$, where $\alpha = 1 - K/K_s$ is the Biot's coefficient ($\phi \leq \alpha \leq 1$). The bulk moduli of the elastic material forming the microporous medium and of the empty (or drained) microporous medium are denoted as K_s and K, respectively.
- $\frac{1}{M} = \frac{\alpha-\phi}{K_s} + \frac{\phi}{K_f}$ is the Biot's bulk modulus. Here K_f is the bulk modulus of the saturating fluid.
- $\mathbf{c}$ is the effective elastic tensor of the microporous skeleton (i.e. dry or "drained") that respects the ellipticity condition ($\exists a > 0 \,/\, \forall \mathbf{e}\,;\, \mathbf{e{:}c{:}e} \geq a\,\mathbf{e{:}e}$) and both the minor symmetry ($c_{ijkl} = c_{jikl} = c_{ijlk}$) and major symmetry ($c_{ijkl} = c_{klij}$).
- For the isotropic case $\mathbf{c}$ reduces to the Lamé coefficients λ and μ. These are related to the bulk modulus and Poisson ratio as $K = \lambda + 2\mu/3$ and $\nu = \lambda/2(\lambda + \mu)$, respectively. Other parameters to be used are the "consolidation" bulk modulus B, defined through $\frac{1}{B} = \frac{1}{M} + \frac{\alpha^2}{\lambda+2\mu}$, and the short term parameters (or "undrained") $\lambda_\infty = \lambda + \alpha^2 M$, $K_\infty = \lambda_\infty + 2\mu/3$, and $\nu_\infty = \lambda_\infty/2(\lambda_\infty + \mu)$.
- The solid displacement of the microporous medium is denoted as $\underline{u}$ while the mean fluid displacement within the volume of the micropores as $\underline{u}_f$. The Darcy flux is defined as $\underline{q} = \phi(\underline{\dot{u}}_f - \underline{\dot{u}}) = i\omega\phi(\underline{u}_f - \underline{u})$.
- $\mathbf{e}(\underline{u})$ is the strain tensor. The tensor of total stress, the tensor of effective stress (i.e. mean stress in the solid skeleton), and the interstitial pressure are denoted as $\boldsymbol{\Sigma}$, $\boldsymbol{\sigma} = \mathbf{c} : \mathbf{e}(\underline{u})$, and p, respectively.
- η is the fluid viscosity and $\mathbf{K}$ is the tensor of intrinsic permeability. Note that the frequency is assumed sufficiently low so that the inertial effects at the micropore scale are negligible. For isotropic porous medium $\mathbf{K} = K\mathbf{I}$.

Pores

- ϕ_p is the porosity related to the pores volume only, i.e. as if the microporous domain were impervious. The total porosity of the media is given by $\boldsymbol{\Phi} = \phi_p + (1 - \phi_p)\phi$. A parameter to be used is $\beta = \sqrt[3]{\phi_p}$.

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- $\underline{u}_p$ and $\underline{\dot{u}}_p = i\omega\underline{u}_p$ denote the fluid displacements and velocity in the pores respectively. The strain rate tensor is given by $\mathbf{D}(\underline{\dot{u}}_p) = i\omega\mathbf{e}(\underline{u}_p)$.
- $\boldsymbol{\sigma}_p$ and p_p stand for the stress tensor and the pressure of the fluid in the pores respectively.

1. Introduction

Double porosity deformable media are characterized by two interconnected networks of fluid-saturated pores of very different sizes that present two very different permeabilities. Such media with hierarchical morphology can be found in either natural materials (e.g. fractured porous reservoirs of first importance in petroleum engineering, clayed sands in civil engineering, biological materials as bones or wood) or in hand made materials (e.g. soils treated by arrays of draining piles in geomechanics, or perforated foams used in acoustics). Physically, the description of such media departs from the usual Biot one (Biot, 1956). As a matter of fact, conversely to single porosity media (Auriault, 1980), the contrast of permeability induces a non-uniform field of pressure within the Representative Elementary Volume (REV) in which the pressure in the pores is different than in the micropores. This situation results generally in two fields of pressure in interaction that macroscopically leads to an enriched Biot description.

The homogenization method (Sanchez-Palencia, 1980) provides a rigorous way to derive the macroscopic description of materials with highly contrasted properties at the microscopic scale. By principle, this method allows linking the local morphology and physics to the macroscopic description. The models developed in Auriault (1983); Arbogast (1989); Hornung and Showalter (1990) explicitly up-scale media presenting a high contrast of thermal conductivity or permeability respectively.

In the works (Auriault and Boutin, 1992, 1993, 1994; Royer and Auriault, 1994), the elastic deformation of the solid frame is taken into account and the permeability contrast is addressed by considering a hierarchical morphology of the pores. Instead of considering such a type of morphology, the homogenization of a Biot poroelastic material presenting fractures, whose size is much larger than the characteristic size of the micropores, has been investigated in Murad et al. (2001); Boutin and Royer (2015). In addition, Rohan et al. (2012) developed a homogenised model for fluid-saturated deformable dual porosity media using the periodic unfolding homogenisation method, while contrasted patchy poroelastic media has been investigated in Pride and Berryman (2003).

A common result of these works is evidencing the existence of two interacting pressure fields, i.e. the pressure in the pores (or fractures) that is locally homogeneous, as in single porosity media, and the pressure in the micropores that diffuses inhomogeneously. Furthermore, it is shown that the pressure field in the micropores is forced by the pressure in the pores (or fractures). This situation leads to the following consequences:

(i) the two pressures fields are constitutively related and cannot be considered independently,
(ii) the inhomogeneous state in the microporous domain induces a *non-instantaneous* flow from the micropores to the pores that gives rise to a distinct attenuation mechanism that does not occur in single porosity media. Such a phenomenon is accounted for by the effective *viscoelastic* properties of the microporous matrix at the macro scale.
(iii) the double porosity model reduces to two different poroelastic Biot models at long and short time that correspond to quasi-instantaneous and vanishing flow respectively (Boutin and Royer, 2015).

This analysis points out the key role of the permeability contrast resulting from the hierarchical morphology. Such a double porosity model has been applied to acoustics of rigid porous media (Olny and Boutin, 2003) and to mass transfer in unsaturated media, and in both contexts the theory has been validated experimentally (Venegas and Umnova, 2011; Ngoc et al., 2007).

The above-cited “homogenized double porosity model” differs from the “phenomenological double porosity model” based on mixture approaches. The latter *postulates* directly at the macroscopic scale the co-existence of two pressure fields in *instantaneous* interaction. Among the literature in this research line, one can mention the pioneering work (Barenblatt et al., 1960) within the framework of rigid porous media, later on modified in Warren and Root (1963) to account for the contrasted permeabilities of the two pore networks. These models were directly extended to poroelasticity in Wilson and Aifantis (1982); Bai et al. (1993); Berryman and Wang (1995), among others. The drawback of these works is that the microstructure is not treated explicitly but appears indirectly through the effective parameters of the postulated global description. As a consequence, the applicability of the model for media with a hierarchical morphology presenting highly contrasted permeabilities is *conjectured but not demonstrated*. In fact, it was shown in Boutin and Royer (2015) that the usual phenomenological assumption of *instantaneous* interaction applies for media constituted by two pore networks (not necessarily highly contrasted) separated by a thin layer of significantly lower permeability, which is a microstructure that departs significantly from that of double porosity media. It is from the difference in the considered morphology that the difference in the models arises. In particular, the assumption of “instantaneous” interaction prevents to capture the viscoelastic features of the effective parameters of double porosity media.

The present work aims to assess the effective parameters of double porosity isotropic media paying particular attention to the effective viscoelastic effect. To do so the following steps are considered:

- firstly, a methodology is developed that allows to reformulate the homogenization results in a framework that enables the analytical estimation of the effective parameters;
- secondly, this approach is applied to two basic double porosity morphologies: (i) large pores embedded in a microporous domain and (ii) a microporous domain with a small amount of highly permeable fractures. In both cases the following four isotropic effective parameters are determined: the bulk and shear moduli, the compressibility, and the pressure-deformation coupling factor;
- thirdly, a parametric study of the analytical results leads to identify for each effective parameter the expected magnitude of the viscoelastic effect and the corresponding characteristic frequency according to: (i) the type of morphology, (ii) the porosity and the characteristic dimension of the Representative Elementary Volume, and (iii) the values of the physical parameters of the solid and fluid constituents.

The paper is organized as follows. In Section 2 the local and macroscopic descriptions are presented. Section 3 summarizes the homogenization approach that allows determining the effective parameters from the local problems. Section 4 is devoted to transposing the homogenization results to a self-consistent framework. Sections 5–7 describe the determination of the effective parameters for the morphology with embedded pores. Section 8 presents the same process but for the morphology with fractures. In both cases the results are illustrated and the features of the parameters are discussed. The main theoretical outcomes and their practical consequences are synthesized in the conclusion. Details of the resolution concerning the effective shear modulus are presented in Appendix A and Appendix B.