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Statistical characterization of the anisotropic strain energy in soft materials with distributed fibers

Alessio Gizzi[a], Anna Pandolfi[b,*], Marcello Vasta[c]

[a] Department of Engineering, University Campus Bio-Medico of Rome, Via A. del Portillo 21, Rome, 00128, Italy
[b] Dipartimento di Ingegneria Civile ed Ambientale, Politecnico di Milano, Piazza Leonardo da Vinci 32, Milano, Italy
[c] Dipartimento INGEO, Università di Chieti-Pescara, Viale Pindaro 42, Pescara, Italy

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ABSTRACT

We discuss analytical and numerical tools for the statistical characterization of the anisotropic strain energy density of soft hyperelastic materials embedded with fibers. We consider spatially distributed orientations of fibers following a tridimensional or a planar architecture. We restrict our analysis to material models dependent on the fourth pseudo-invariant I_4 of the Cauchy–Green tensor, and to exponential forms of the fiber strain energy function Ψ_{aniso}. Under different loading conditions, we derive the closed-form expression of the probability density function for I_4 and Ψ_{aniso}. In view of bypassing the cumbersome extension–contraction switch, commonly adopted for shutting down the contribution of contracted fibers in models based on generalized structure tensors, for significant loading conditions we identify analytically the support of the fibers in pure extension. For uniaxial loadings, the availability of the probability distribution function and the knowledge of the support of the fibers in extension yield to the analytical expression of average and variance of I_4 and Ψ_{aniso}, and to the direct definition of the average second Piola–Kirchhoff stress tensor. For generalized loadings, the dependence of I_4 on the spatial orientation of the fibers can be analyzed through angle plane diagrams. Angle plane diagrams facilitate the assessment of the influence of the pure extension condition on the definition of the stable support of fibers for the statistics related to the anisotropic strain energy density.

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1. Introduction

In the last two decades soft tissue biomechanics and advanced constitutive modeling have been experiencing a growing research activity. The outcomes of this expanding impulse are glaring, since computational models of biological materials are now commonly used in tissue engineering design and development. Among others, well recognized examples of application can be found in cardiovascular functioning (Driessen et al., 2005), haemodynamics (Horgan and Saccomandi, 2003; Li and Robertson, 2009; Tsamis et al., 2013), damage and remodeling (Ferrara and Pandolfi, 2008; Ni Annaidh et al., 2012; Sánchez et al., 2014). As a consequence of the intrinsically *patient-specific* nature and of the microstructural complexity of biological tissues, their modeling is very challenging and still incomplete. The main difficulties are related to highly nonlinear behaviors and to inhomogeneities in the mechanical properties (Sacks, 2003).

Computational approaches for modeling the constitutive relations of biologic soft materials exhibiting reversibility rely on the definition of an appropriate strain energy density, eventually embedding, in a continuum sense, the underlying multiscale structure of the material. Within this perspective, microstructural constitutive models account for the

* Corresponding author. Tel.: +39 223994217.
E-mail address: anna.pandolfi@polimi.it (A. Pandolfi).

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List of symbols

$\mathbf{a}$:	fibers unit vector
$\mathbf{A}$:	fibers structure tensor
$\mathbf{H}$:	average fibers structure tensor
$\mathbb{H}$:	average fourth order structure tensor
Θ, Φ:	aleatoric Euler angles
θ, ϕ:	occurrence of the aleatoric Euler angles
I_4:	aleatoric fourth pseudo-invariant
I_4:	occurrence of the aleatoric fourth pseudo-invariant
λ:	imposed stretch
Ψ:	aleatoric anisotropic isochoric strain energy density
Ψ:	occurrence of the aleatoric anisotropic isochoric strain energy density
$\langle\mathbf{S}\rangle$:	average anisotropic stress tensor
$d\omega$:	spherical solid angle
$d\theta$:	planar angle increment
Ω:	unit sphere integration domain
$\mathcal{D}$:	generic integration domain
$\mathcal{D}_{\mathrm{F}}$:	extension–contraction integration domain
$\mathcal{D}_{\mathrm{E}}$:	pure-extension integration domain
$\rho(\mathbf{a})$:	generic probability distribution function
$\rho_{\Theta}(\theta)$:	probability distribution function of Θ
$\rho_{\mathrm{I}_4}(I_4)$:	probability distribution function of I_4
$\rho_{\Psi}(\Psi)$:	probability distribution function of Ψ
N_{Θ}:	normalization factor of $\rho_{\Theta}(\theta)$
N_{I_4}:	normalization factor of $\rho_{\mathrm{I}_4}(I_4)$
N_{Ψ}:	normalization factor of $\rho_{\Psi}(\Psi)$
I_4^*:	average fourth invariant
Ψ^*:	strain energy density evaluated at the average invariant $\Psi = \Psi(I_4^*)$
$\sigma_{I_4}^2$:	variance of I_4
σ_{Ψ}^2:	variance of Ψ
PDF:	probability distribution function

architecture and the spatial organization of the material structure by introducing explicitly their description in the strain energy density. A microstructural approach permits to better understand the physical significance of the material constants of the tissue, facilitating the achievement of a correct thus predictive macroscopic material model to be used in numerical applications.

To clarify the nature of the variability in the mechanical properties of fiber-reinforced soft tissues, Lanir (1983) introduced a stochastic approach within the definition of constitutive models. Lanir defined the strain energy density as the integral of the strain energy density of single fibers, spatially oriented according to a statistical distribution. Extensions and particular applications of this approach have been discussed in subsequent research (Holzapfel et al., 2000; Rodríquez et al., 2006; Alastrué et al., 2007; Federico and Gasser, 2010; Gizzi et al., 2014).

In spite of the large literature flourished from the seminal work of Lanir, we can acknowledge only a few attempts of characterizing analytically the statistical properties of the probability distribution functions (PDF) of complex materials showing an anisotropic microstructure. In particular, Zulliger et al. (2004) considered a log-logistic PDF for the progressive engagement of the fibers, while more recently Rodríquez et al. (2006) introduced a stochastic structural model describing the waviness of a fiber bundle. The material model described in Rodríquez et al. (2006), derived from the worm-like chain model of Arruda and Boyce (1993), adopts a PDF of Beta type, calculated using Bayesian statistics but assuming a deterministic orientation of the fibers.

This study aims at characterizing analytically the statistics of mechanically significant quantities related to soft materials embedded with a stochastic distribution of reinforcing fibers. The presence of dispersed fibers confers to the medium a certain degree of anisotropy not easy to be described or quantified, whereas the availability of handy parameters would be highly desirable, especially in numerical applications. We consider hyperelastic materials, and restrict our consideration to isochoric behaviors. We assume that the anisotropic behavior of the material can be fully described by the fourth isochoric pseudo-invariant $\bar{I}_4$, which measures the square of the stretch in the direction of the fibers. Starting from a well established theoretical framework (Gasser et al., 2006; Pandolfi and Vasta, 2012; Vasta et al., 2014), we assume the tridimensional distribution of reinforcing fibers to be defined through of the composition of two PDFs associated to the Euler angles Θ and Φ, regarded as aleatoric variables. For uniaxial loading, we derive analytically the closed-form PDF of $\bar{I}_4$, as sole aleatoric variable defining the distribution, and, correspondingly, the PDF of the anisotropic strain energy density, Ψ_{aniso}. We identify the theoretically correct ranges of fiber in extension in terms of the meridian angle Θ for $\bar{I}_4$ and Ψ_{aniso}, by generalizing the approximate estimate recently proposed in Holzapfel and Ogden (2015), and we provide a better approximation of the average second Piola–Kirchhoff stress tensor. Furthermore, we discuss the implication of multiaxial loading on the range of fibers in extension, for tridimensional and planar distributions. We provide analytical forms of the PDFs and of their support for uniaxial and shear loadings, improving the computational efficiency of the stability condition for compressed fibers exclusion. For more general loadings, we illustrate how, from the observation of angle plane plots, it may be possible to define the range of fibers in extension, to be considered in the evaluation of the mechanically relevant statistics of the material.

The paper is organized as follows. In Section 2 we formulate the generalities of the material models for distributed fibers considered in this study and introduce the approximations for the strain energy density and stress tensor. In Section 3 we derive the closed-form PDF for the fourth pseudo-invariant and the anisotropic free energy density in the particular case of uniaxial loading in the direction of the fibers. More general loading conditions for tridimensional distributions of fibers are discussed in Section 4. In Section 5 we derive the PDFs for planar distributions of fibers. In Section 6 we present quantitative comparison between the mechanical response of our novel closed-form derivations and of alternative previous models. The results are discussed in Section 7. Limitations and future perspectives are drawn in Section 8.

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