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Adhesive impact of an elastic sphere with an elastic half space: Numerical analysis based on the method of dimensionality reduction

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ABSTRACT

An impact of an elastic sphere with an elastic half space in the presence of adhesion is studied numerically using the method of dimensionality reduction. It is shown that the rebound velocities and angular velocity, written in proper dimensionless variables, are determined by a function of only the ratio of tangential and normal stiffness ("Mindlin-ratio") and one further parameter describing the adhesion properties of the contact. The obtained numerical results can be approximated by analytical expressions.

1. Introduction

Impacts of solid particles are of interest for many physical and technological processes related to the dynamics of granular media (Thornton and Yin, 1991; Ciamarra et al., 2004; Jop et al., 2006; Brilliantov et al., 1996). Even for spherical particles from purely elastic material, the detailed dynamics of the impact can be very complicated.

However, the problem formulation can be simplified by using the Method of Dimensionality Reduction (MDR) (Popov and Psakhie, 2007; Popov; 2013; Popov and Heß, 2014, 2015). In the MDR, the contact problem of a particle with a half space is replaced by a contact of a plain indenter of properly modified profile shape with a linear elastic foundation consisting of independent springs. This simplifies the contact problem drastically and opens new ways for analytical and numerical treatment of dynamic normal and tangential contacts.

In the recent publication (Lyashenko and Popov, 2015) an impact of an elastic sphere with an elastic half space under no-slip conditions was studied numerically using the MDR. It was shown that the rebound velocity and angular velocity, written in proper dimensionless variables, are determined by a function of only the ratio of tangential and normal stiffness ("Mindlin-ratio"). However, in many real situations, for example powder technologies or in biological systems, the contact force between particles during their interactions has adhesive contribution. In the present paper, the solution obtained in Lyashenko and Popov (2015) will be extended to the case of adhesive impacts, under the same assumptions as in the theory by Johnson, Kendall and Roberts in Johnson et al. (1971), known as JKR – theory.

In the experimental work by Waters and Guduru (2009), it has been shown that normal and tangential contributions to adhesion forces may depend on the direction of the

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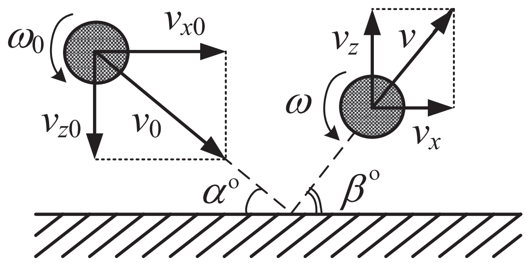

Fig. 1. Schematic representation of an impact.

motion of the particle. In this paper we assume conditions under which the influence of tangential loading on the adhesion in the normal direction can be neglected. In this case we can describe the influence of adhesion by a standard procedure within the framework of the MDR (Popov and Heß, 2014). Kosinski and Hoffmann (2011) studied the influence of adhesion in Lagrangian particle-fluid flows using the JKR – theory. They gave dimensionless formulations and comprehensive solutions for a wide parameter space in the case of a pure normal adhesive impact. (Ledvinkova and Kosek (2013) used discrete element method (DEM) to study impacts between polymer particles, which they modeled as agglomerations of elastic microelements bound by adhesive forces. They focused on impact times and the processes of agglomeration and disintegration of the polymer particles during the impact. The impact problem for the adhesive normal contact was studied analytically and experimentally by Andres (1995). He thereby applied the adhesion models of Johnson et al. (JKR – theory), Deryagin et al. (DMT – theory) and Maugis. A comprehensive analytical model of the adhesive normal contact impact problem using the JKR – theory was derived by Brilliantov et al. (2007). The oblique impact without adhesion was studied in a series of papers by Maw et al. (1976, 1977, 1981). To our knowledge no efforts have been published yet to address the oblique adhesive impact problem.

The paper is organized as follows. In Section 2, we first give the problem formulation and after that in Section 3 reproduce, as a basis for further consideration, the classical solution for an impact without adhesion based on the assumption of rigid rotation at the last moment of contact. We then solve a simplified model with adhesion under assumption of constant contact stiffness. This derivation provides the general form of the solution and the main dimensionless parameters which will play a role in the subsequent rigorous simulation. In Section 4, the numerical model of the impact problem based on the MDR is presented, the results of which are described in Section 5 and compared with experimental results for the normal adhesive contact in Section 6. The final Section 7 concludes the paper.

2. Problem formulation and assumptions

Let us consider an impact of an elastic sphere with mass m and radius R on an elastic half space, as shown in Fig. 1. Let the moduli of elasticity of the sphere and the half space be E_1 and E_2, their Poison's numbers ν_1 and ν_2, and their shear moduli G_1 and G_2, accordingly. We will restrict ourselves to the case of elastically similar media, i.e.

$$\frac{1-2\nu_1}{G_1}=\frac{1-2\nu_2}{G_2}, \tag{1}$$

for which the tangential and normal contact problem will decouple. We will work within the half space approximation, so the indentation depth is always assumed to be small compared to the contact radius. Moreover all lengths associated with the contact are considered negligible compared to the sphere radius R. We assume that the adhesion in the contact can be described by the JKT-type theory in spite of the fact that no-slip condition is assumed in the whole contact. Note, that it was shown by Borodich et al. (2012) that in the case of intermolecular attraction the assumption of no-slip is physically more appropriate than the one of no shear stresses in the contact, usually used within the JKR – theory. However, in the case of elastic similar materials satisfying Eq. (1), there is no difference between the frictionless and no-slip normal contact. And even in the case of materials which do not satisfy (1), the following theory can be used in a very good approximation as the difference of the results of an adhesive contact with and without sliding in the contact interface is of the same order of magnitude as the difference of normal stiffness of a cylindrical indenter with and without friction, which for Poisson numbers between 0.3 and 0.5 does not exceed 3% (Borodich et al., 2012).

We will also assume a quasi-static impact, i.e. impact velocities being much smaller than the velocity of sound in the elastic half space. In this case the inertia of the half space is negligible. The loss of kinetic energy during the impact due to elastic waves propagating into the half space (Hunter, 1957) is neglected.

The main notations are illustrated in Fig. 1: The incident velocity of the center of mass of the sphere is v_0 with horizontal and vertical components v_{x0} and v_{z0}, the incident angular velocity ω_0, the rebound velocity is v with components v_x and $-v_z$, the grazing angle is $\alpha°$, and the rebound angle $\beta°$.

3. A linear model of the oblique adhesive impact without slip

Let F_x and F_z be the components of the contact force acting on the sphere during the impact. The equations of motion of the sphere in the integral form can be written as

$$m(v_z-v_{z0})=-\int_0^t F_z(t')\mathrm{d}t', \tag{2}$$

$$m(v_x-v_{x0})=-\int_0^t F_x(t')\mathrm{d}t', \tag{3}$$

$$I(\omega-\omega_0)=-R\int_0^t F_x(t')\mathrm{d}t', \tag{4}$$

where t denotes the duration of the impact and $I=(2/5)mR^2$ is the moment of inertia of the sphere. We first reproduce the classical text-book solution of the impact problem. Together with the rolling condition for the tangential rebound velocity,

$$v_x+\omega R=0, \tag{5}$$