Mechanics of Materials 92 (2016) 261–274

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

The effect of fiber misalignment on the homogenized properties of unidirectional fiber reinforced composites

Yujun Li[a,*], Bertram Stier[a], Brett Bednarcyk[b], Jaan-Willem Simon[a], Stefanie Reese[a]

[a] Institute of Applied Mechanics, RWTH Aachen University, Mies-van-der-Rohe Str. 1, D-52074 Aachen, Germany
[b] NASA Glenn Research Center, Cleveland, Ohio, U.S.A.

A R T I C L E I N F O

Article history:
Received 11 November 2014
Revised 12 July 2015
Available online 20 October 2015

Keywords:
Unidirectional composites
Fiber misalignment
Orientation distribution function
Microsphere
Homogenization

A B S T R A C T

Unidirectional fiber-reinforced composites have been widely used over the past several decades in industry due to their high specific strength and superior fatigue characteristics. In order to predict their overall mechanical properties, typically, a homogenization procedure is used to relate the constituent properties and the macroscopic behavior, in which the representative one-dimensional material description is generalized to a fully three-dimensional constitutive model. The aim of this study is primarily to understand the influence of fiber misalignment on the effective composite material properties. In order to achieve this, a microsphere based homogenization approach is proposed, in which the passage from microstructural contributions to the macroscopic response is obtained by integration over the surface of a unit microsphere. The result is compared with a micromechanically motivated model and the High-Fidelity Generalized Method of Cells model. The results illustrate the effects of the fiber misalignment degree in terms of the concentration dependence of the predicted overall properties. From these findings, elastic properties can be obtained for the design of composite structures.

1. Introduction

Fiber reinforced composite materials are commonly used in the aerospace, automotive, marine, and construction industries due to their high strength, durability, corrosion resistance and damage tolerance characteristics. They also provide high adaptability to different situations because the component materials can be tailored to meet the design requirements. For most practical analysis of composite structures, effective material properties are used instead of the individual constituent properties and their geometrical arrangements. However, these analyses may not be sufficiently accurate if they are performed only at the macroscopic scale. On the other hand, in most cases practical problems are too complex and time consuming if handled at microscopic scale only. A compromise approach is what is well known as multiscale modeling, whose basic task is to design combined macroscopic-–microscopic computational methods that are much more efficient than solving the full microscopic model, but at the same time give the information with the desired accuracy (E et al., 2007; Kanouté et al., 2009).

For fiber reinforced composites, most of the homogenization processes found in the literature deal with fixed-orientation fibers (Aboudi et al., 2013; Stier et al., 2014; Simon et al., 2014). However, it is well known that the macroscopic mechanical properties of fiber reinforced composites can be strongly influenced by the spatial distribution of the fiber alignment and also show a pronounced direction dependence (Basu et al., 2006). Such fiber spatial distribution can often occur due to the production process, such as the resin flow conditions inside the die for mold-injected fiber reinforced polymers (Kugler and Moon, 2002). But usually it is impossible to obtain the exact predetermined

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[*] Corresponding author. Tel.: +49 2418025013; fax: +49 2418022001.
E-mail address: yujun.li@rwth-aachen.de, li.yujun@hotmail.com (Y. Li).

http://dx.doi.org/10.1016/j.mechmat.2015.10.002

spatial orientations of fibers in practice. It is therefore of interest to investigate how the overall properties of composites are affected by the fiber misalignment from nominal orientations. The basic framework to address this problem is first to describe the micro-geometries by use of the orientation distribution function (ODF) of the reinforcements, whose parameters can be determined experimentally. Next, orientated averaging of the appropriate tensors is performed, which, in general, can be done via direct numerical integration (Pettermann et al., 1997; Doghri and Tinel, 2005; Gasser et al., 2006; Alastrué et al., 2009).

This kind of orientation averaging procedure has been employed within several different methods, namely, the Mori–Tanaka mean field approach (Benveniste, 1987), the High-Fidelity Generalized Method of Cells (HFGMC) (Aboudi et al., 2013), and the microsphere model. Pettermann et al. (1997) adopted the Mori–Tanaka micromechanics method to predict the overall thermoelastic properties of two-phase short fiber reinforced composites described by truncated Gaussian distributions. Their study also provided information about the influence of the fiber orientation distribution on the effective properties and the basic mechanisms of interaction between the constituents. Bednarcyk et al. (2014) performed orientational averaging of the concentration tensors within the HFGMC micromechanics model to predict the effective properties and initial damage surfaces of composites with a Gaussian distribution of fiber orientations.

An alternative approach for predicting the mechanical behavior of dispersed materials is called the microsphere model, also denoted as microplane method, which was originally proposed by Bažant and Oh (1985). Miehe et al. (2004) extended this method to finite deformations and applied it first to rubber-like materials. Menzel and Waffenschmidt (2009) used the model, with an evolving orientation distribution function, to simulate remodeling in soft tissues. Murtada et al. (2010) applied the microsphere approach to analyze smooth muscles, wherein a specialized distribution of the muscle contractile fiber orientation, as a function of stretch, was employed. Alastrué et al. (2009; 2010) used a π -periodic von Mises orientation distribution function, and then a Bingham orientation distribution function, in applying the microsphere model to blood vessels. Waffenschmidt et al. (2014) have recently incorporated the microsphere approach into a non-local gradient-based damage model for composites. However, this kind of approach has not been applied to the carbon fiber and jute fiber reinforced composites considered in this work.

In the present investigation, the microsphere approach is employed to predict the effective behavior of unidirectional composites, wherein the fibers are assumed to follow a von Mises orientation distribution. Further, the effects of fiber misalignment on the overall mechanical properties are fitted to experiments. The great advantage of the microsphere approach is to develop a sufficient approximation of three-dimensional material behavior by only considering the uniaxial one. Meanwhile, a micromechanically motivated model proposed by Reese et al. (2001); Reese (2003) and the High-Fidelity Generalized Method of Cells (HFGMC) (Aboudi et al., 2013) are adopted for comparison and validation, along with experimental data.

The remainder of this paper is organized as follows. In Section 2 a brief description of the microsphere methodology is provided, followed by details on the incorporation of the statistical fiber orientation distributions and the analytical calculation of initial elasticity tensor. Section 3 illustrates Reese's anisotropic material model while in Section 4 the HFGMC micromechanical model is described. Tension tests performed on carbon fiber reinforced unidirectional composites are described in some detail in Section 5. The results and discussion are presented in the final section.

2. Continuum mechanical model based on microsphere approach

To model the anisotropic and hyperelastic constitutive response of composites, a method is to establish the Helmholtz free-energy function $\psi = \psi(\mathbf{C})$, defined per unit reference volume, which is also referred to as a strain-energy density function (SEDF). Using the SEDF, the relation between the second Piola–Kirchhoff stress tensor **S** and the right Cauchy–Green tensor **C** is given by

$$\mathbf{S} = 2\,\frac{\partial\psi(\mathbf{C})}{\partial\mathbf{C}} \tag{1}$$

The basic concept of the microsphere approach is first to define one-dimensional material models, which coincide with the representative material directions, and then the overall mechanical behavior is determined by averaging the energy density over different space orientations. Accordingly, the first step is to find the one-dimensional constitutive models aiming to reflect the basic mechanical properties of the composite that result from its internal constitution, which is addressed in Section 2.1. Then, the homogenization procedure follows to describe the macroscopic response as discussed in Sections 2.2 and 2.3. In Section 2.4, the initial elasticity tensor is calculated analytically. The final section gives a brief discussion of the discretization on the unit sphere.

2.1. One-dimensional material models

From the microstructure characteristics of unidirectional fiber reinforced composites, it is obvious that there exist two representative directions, namely the longitudinal direction and the transverse direction. From the rheological point of view, the longitudinal direction ($\parallel$) could be considered as a parallel connection of fiber and matrix in terms of Voigt, while in the transverse direction ($\perp$), a series connection of fiber and matrix is set according to Reuss. In this case, the one-dimensional SEDF for elastic material can be represented by

$$\psi_{\parallel} = \frac{1}{2}E^{\parallel}(\lambda-1)^2, \qquad \psi_{\perp} = \frac{1}{2}E^{\perp}(\lambda-1)^2 \tag{2}$$

in which

$$E^{\parallel}(\lambda) = \varphi E_F(\lambda) + (1-\varphi)E_M(\lambda)$$

$$E^{\perp}(\lambda) = \left[\varphi\frac{1}{E_F(\lambda)} + (1-\varphi)\frac{1}{E_M(\lambda)}\right]^{-1} \tag{3}$$

are the Young's moduli in longitudinal and transverse direction, respectively. The variable φ denotes the fiber volume fraction, and E_F and E_M denote the Young's moduli of the fiber