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Stress-deformable state of isotropic double curved shell with internal cracks and a circular hole

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ABSTRACT

This work is devoted to the stress–strain state of isotropic double curved shell with defect system. The construction is weakened by two non-through thickness (internal) cracks of different length and by a circular hole located between cracks. In this study we use the line-spring model. Within the framework of this model cracks are modeled as mathematical cuts of shell's middle surface. This leads to a two-dimensional problem. The problem is reduced to a system of eight boundary integral equations. To ensure the uniqueness of solution an additional equation is added. In the numerical solution of the problem special quadrature formulas for singular integrals of Cauchy type and the finite difference method are applied. The influence of defects on each other for double curved shell has been investigated. The given theoretical results can be used for the calculation of structural elements with holes, cracks on the strength and fracture toughness in various branches of engineering.

1. Introduction

Thin-walled structures such as shells and plates are used in modern engineering, shipbuilding, aerospace and other industries. Often such structures have defects like holes, cuts and inclusions which are the stress concentrators. The definition of internal forces in the vicinity of the defect under certain loads is an important task of fracture mechanics.

A considerable attention is paid to the material's behavior in bodies containing multiple defects. Their interaction can ambiguous affect the structure deformation behavior and the resource of its operation in service.

Time to time any technical structure has some destruction and the deterioration can be different. Materials in contact with the external environment tend to partially collapse. This process is also influenced by many other factors, such as the quality of the materials, extreme weather conditions, errors of project documentation. It is important in the early stages of failure to stop this process because consequences can be disastrous.

The determination of load distribution in the vicinity of defects allows analyzing the stress–strain state of elastic constructions. The most dangerous defects in the constructions are initiated by cracks and holes.

A number of works has been devoted to mutual interaction of defects in solids. Ishihama (1984) explored the definition of the stress state for cylindrical shell with two axial cracks and a circular hole located between them. Cracks emanating from a hole have been analyzed in works of Akash et al. (2013), Guo and Lu (2009), Guo and Liu (2007), Jones et al. (2013), Laverne et al. (2004), Nishioka et al. (2009), Picazo and Sevostianov (2011), Romantsov and Chernyshenko (1995), Xiangqiao and Baoliang (2012)

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and Yan (2004). Most of these works consider only the plate as a special case of the shell (Guo and Liu, 2007; Laverne et al., 2004; Nishioka et al., 2009; Xiangqiao and Baoliang, 2012; Yan, 2004). Finite element and boundary element methods are often employed to obtain the solution numerically.

In Dovbnya and Krupko (2014a) a shell of double curvature with a circular hole was considered. To ensure the uniqueness of the solution it was necessary to add one more equation. The numerical study was conducted for the case when the differential equation relating the unknown functions was chosen.

This work is a continuation of the study (Dovbnya and Krupko, 2014b), which was focused on the mutual influence of non-through thickness cracks (surface cracks) and a circular hole in an isotropic plate. The research, which is presented in this paper, is devoted to a double curved shell.

2. Formulation

A thin shallow isotropic shell of constant thickness h is considered (Fig. 1). The coordinate system is chosen in such a way that the axes x, y are oriented along the directions of principal curvature of shell middle surface and axis z is directed along its normal. We analyze the shell for double curvature under the action of an external balanced load – uniform tension along the y-axis of intensity p.

The considered construction is weakened by a circular hole of radius r_0 located in the centre of coordinate system as well as by two collinear non-through thickness cracks directed along the axis x with lengths $2l_i$, $i = 1, 2$. The distance between the centres of cracks is $2d$. In this paper we consider internal cracks of semi-elliptical shape, their maximum depth is reached at a central point and is equal to l_{0i}, $i = 1, 2$. Along the contour (x) of an internal crack the depth is given by the following formula

$$L_{0i}(x) = l_{0i}\sqrt{(1-(x/l_i)^2)}, \quad i = 1, 2.$$

We have the following assumption: we analyze small cuts. Due to the assumption that a cut is a small enough, the bending stresses, due to cut, will have a local character. Stress–strain state of the shell in the vicinity of the small hole is slightly different from the corresponding state in the flat plate. In the approximate solution of boundary value problems the middle surface curvatures rely as small parameters (Guz et al., 1980).

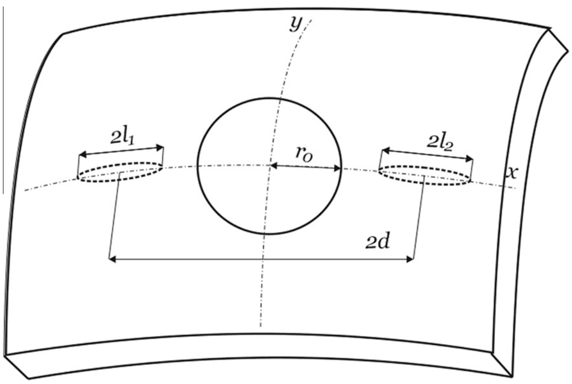

Fig. 1. The shell with defects.

The non-through thickness crack in a shell is a three-dimensional problem. To solve the problem we use the line-spring model which allows us to reduce the three-dimensional problem to the two-dimensional one, yet certain remarks apply. In the solid material layer under and above the front of through thickness cracks (internal cracks), when tensile and bending are acted, the stresses occur, which are reflected by introducing unknown forces $T(i)$ and moments $M(i)$ on the line of the i-crack $(i = 1, 2)$. If external load (forces and moments) is directed to crack opening the unknown forces $T(i)$ and moments $M(i)$ are directed to crack joining. Thus, we have fictitious through thickness cracks instead non-through thickness cracks. We consider that the sizes of cracks and a hole are bigger in comparison with the shell thickness, but smaller than other linear sizes. All these remarks allow us to consider the problem of equilibrium of a thin shell with cracks and a hole within the framework of the two-dimensional shell theory where cracks are modeled as mathematical cuts of shell median surface (Rice and Levy, 1972).

Smooth sections of the hole contour (L_0), cracks (L_1, L_2) are described as

$$L_0 : \alpha_0(t) = r_0\cos(t), \quad \beta_0(t) = r_0\sin(t), \quad t \in [0, 2\pi),$$

$$L_1 : \alpha_1(t) = l_1(t) + d, \quad \beta_1(t) = 0,$$

$$L_2 : \alpha_2(t) = l_2(t) - d, \quad \beta_2(t) = 0, \quad t \in [-1, 1].$$

We assume contours of defects are free from loads. Stress–strain state of the shell in the vicinity of the small hole is slightly different from the corresponding state in the flat plate (Guz et al., 1980).

Thus, boundary conditions for through thickness cracks have the form

$$T_2^1(x) = 0, \quad M_2^1(x) = 0,$$

$$T_2^2(x) = 0, \quad M_2^2(x) = 0,$$

but for fictitious through thickness cracks (non-through thickness cracks), the boundary conditions have the following form

$$T_2^1(x) = -T^1(x), \quad M_2^1(x) = -M^1(x), \tag{1}$$

$$T_2^2(x) = -T^2(x), \quad M_2^2(x) = -M^2(x),$$

where $T_2^1(x)$, $T_2^2(x)$ – unknown membrane forces; $M_2^1(x)$, $M_2^2(x)$ – unknown bending moments; $T^i(x)$, $M^i(x)$, $i = 1, 2$ – are defined from the line spring model (Rice and Levy, 1972) as follows

$$\begin{pmatrix} T^1 \\ \frac{6M^1}{h} \end{pmatrix} = \frac{l_1/h}{(1-\nu^2)} \begin{pmatrix} a_{11}\int_{-1}^{1}\psi_6(t)sign[s-t]dt \\ a_{22}\frac{\sqrt{1-\nu^2}}{\sqrt{3(1-\nu)(3+\nu)}}\int_{-1}^{1}\psi_8(t)sign[s-t]dt \end{pmatrix}, \tag{2}$$