Mechanics of Materials 90 (2015) 118–130

Contents lists available at ScienceDirect

Mechanics of Materials

journal homepage: www.elsevier.com/locate/mechmat

A study of transverse cracking in laminates by the Thick Level Set approach

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ARTICLE INFO

Article history:
Received 11 November 2014
Received in revised form 2 April 2015
Available online 14 May 2015

Keywords:
Non-local damage
Thick Level Set
Transverse cracking
Composite laminates

ABSTRACT

In this paper, we present an analysis of the transverse cracking and interface failure process induced in layered materials (such as composite laminates) subjected to tensile loading, with a new level set based non-local modeling approach for damage growth (TLS: Thick Level Set). In particular, a 2D finite element model is built to study damage in a cross-ply laminate. The study aims at evaluating the capacity of the TLS method to predict evolution of damage at the ply level, including initiation, propagation, merging of cracks or delamination. We show how this numerical model is able to reproduce key features such as crack spacing saturation and other experimental observations.

1. Introduction

The appearance and development of transverse cracks in layered materials is an important problem in many fields of engineering, for example in composite laminates (Garret and Bailey, 1977; Highsmith and Reifsnider, 1982; Manders et al., 1983), thin-films (Thouless et al., 1992), civil engineering (Hong et al., 1997), or geology (Price, 1966). It is well established that in this configuration of layered materials, cracks tend to self-organize, with a spacing which is directly related to the relative thickness of layers. Explanations for this phenomenon, based on shielding effects, have been proposed on the basis of fracture mechanics (e.g. Bai and Pollard (1999) and Bai et al. (2000)). This type of analysis relies on the study of the effect of discrete cracks placed in elastic layers, and although it can explain why a given crack distribution is optimal or natural in some way, it cannot provide details on the process which would lead to this crack distribution.

Transverse microcracking and local delamination in fiber-reinforced composite laminates have been studied mainly within the framework of finite fracture mechanics. Analytical approaches have been used for crack growth with energetic criteria that have allowed the definition of a large number of successful fracture models such as in Dvorak and Laws (1987), and others (Hashin, 1996; Nairn, 2000; Varna et al., 1999). Rebière and Gamby (2004) recently proposed an analytical energetic criterion for modeling crack initiation and propagation in the matrix as well as delamination in cross-ply laminates. Alternatively, numerical models have been proposed to model matrix fracture, based on finite element approaches combined to bulk damage (Berthelot et al., 1996) or cohesive elements (Okabe et al., 2004). The meso-model developed by Ladevèze and Lubineau (2001) and Ladevèze et al. (2006) predicts damage evolution directly at the ply level. Due to the variability of local material properties within the plies, the heterogeneity of mesoscopic structures must be considered. Some authors introduced a statistical criterion in strength like in Berthelot and Le Corre (2000), or a statistical criterion in toughness as in Andersons et al. (2008). Numerical studies of the propagation of debonding

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http://dx.doi.org/10.1016/j.mechmat.2015.05.003

at the tip of transverse cracks were also treated by a variational approach to fracture (Baldelli et al., 2011).

Modeling the progressive degradation of materials from an initial undamaged state to total failure is still a challenge in computational mechanics. Damage models can be used to describe the initial degradation of mechanical properties, while fracture mechanics is well adapted to final stages leading to fracture. The Thick Level Set (TLS) approach, proposed by Moës et al. (2011), allows for a seamless transition between damage and fracture, while providing the necessary regularization in presence of softening.

In this paper, we present an analysis of the transverse cracking and interface failure process induced in layered materials (such as composite laminates) subjected to tensile loading, using the TLS approach. The main aspects of this method will be presented in Section 2. In Section 3, a 2D finite element model is built to study damage in a cross-ply laminate. The study aims at evaluating the capacity of the TLS method to predict evolution of damage at the ply level, including initiation, propagation, merging of cracks or delamination. For this, we will start from undamaged (virgin) material with small but random variations of mechanical properties, and simulate the initiation and evolution of cracks. In Section 4, we show how this numerical model is able to reproduce key features such as crack spacing saturation and other experimental observations. We also study the effect of some algorithmic parameters involved in the TLS method. The paper closes with some conclusions and perspectives.

2. Thick Level Set (TLS) approach

2.1. Continuum damage

Our objective in this work is to study the development of transverse cracks in layered materials, starting from a virgin (crack-free) state. It is nowadays well established that continuum damage models (CDM) are appropriate to treat early stages of material degradation. Abundant literature on CDM is available, which analysis is nonetheless beyond the scope of the present paper, and we will simply refer the reader to Lemaître et al. (2009).

2.1.1. Local constitutive relations

We will work under assumptions of linearized kinematics, and consider a simple elastic-damage model described by

$$\boldsymbol{\sigma} = \mathbb{C}(d) : \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\boldsymbol{\varepsilon}$ the engineering strain tensor, and $\mathbb{C}(d)$ a fourth-order elasticity tensor, function of the scalar damage variable $d \in [0, 1]$. The model can alternatively be described in the framework of generalized standard materials (Halphen and Nguyen, 1975; Germain et al., 1983), by defining the free energy potential, a function of the material state $\{\boldsymbol{\varepsilon}, d\}$:

$$W(\boldsymbol{\varepsilon}, d) = \frac{1}{2}\boldsymbol{\varepsilon} : \mathbb{C}(d) : \boldsymbol{\varepsilon}. \tag{2}$$

This potential in turn allows to define thermodynamical forces conjugate to state variables $\boldsymbol{\varepsilon}$ and d:

$$\boldsymbol{\sigma} = \frac{\partial W}{\partial \boldsymbol{\varepsilon}}(\boldsymbol{\varepsilon}, d) \tag{3}$$

$$Y = -\frac{\partial W}{\partial d}(\boldsymbol{\varepsilon}, d) \tag{4}$$

It is easily verified that (3) is equivalent to (1). Relation (4) defines the energy release rate Y, thermodynamically conjugate to damage d. The equation describing the evolution of damage is then obtained through the dissipation potential $\psi(Y)$:

$$\dot{d} \in \partial_Y \psi(Y) \tag{5}$$

where $\partial_Y \psi$ denotes the sub-gradient of $\psi(Y)$, a (potentially non-regular) convex function of Y. Following standard arguments, convexity of $\psi(Y)$, combined to conditions $\psi(0) = 0$ and $\psi(Y) \geqslant 0 \forall Y$, ensures positivity of the dissipation:

$$\mathcal{D} = Y\dot{d} \geqslant 0. \tag{6}$$

This formalism allows to simultaneously cover both rate-independent and rate-dependent damage models. Indeed, the rate-independent case corresponds to a lower semi-continuous dissipation potential of the form:

$$\psi(Y) = \begin{cases} 0 & \text{if } Y \leqslant Y_c, \\ +\infty & \text{if } Y > Y_c \end{cases} \tag{7}$$

and (5) is then equivalent to Karush–Kuhn–Tucker conditions: $\dot{d} \geqslant 0,\ Y - Y_c \leqslant 0,\ (Y - Y_c)\dot{d} = 0$. The rate-dependent case corresponds to more regular functions (e.g. power-law expressions of Y or $Y - Y_c$). Finally, a dual dissipation potential ψ^* can be defined through a Legendre–Fenchel transform:

$$\psi^*(\dot{d}) = \sup_Y \left[Y\dot{d} - \psi(Y)\right] \quad \text{and} \quad Y \in \partial_{\dot{d}}\psi^*(\dot{d}) \tag{8}$$

where convexity of $\psi^*(\dot{d})$ is guaranteed by properties of Legendre transforms.

2.1.2. Boundary-value problem

The boundary-value problem at a given time t then consists in the static mechanical balance equation

$$\nabla \cdot \boldsymbol{\sigma} + \boldsymbol{b} = 0 \quad \forall \boldsymbol{x} \in \Omega \tag{9}$$

together with boundary conditions

$$\boldsymbol{u} = \bar{\boldsymbol{u}} \quad \forall \boldsymbol{x} \in \partial_u\Omega \tag{10a}$$

$$\boldsymbol{\sigma} \cdot \boldsymbol{n} = \bar{\boldsymbol{t}} \quad \forall \boldsymbol{x} \in \partial_\sigma\Omega \tag{10b}$$

where $\partial_u\Omega \bigcup \partial_\sigma\Omega = \partial\Omega$ and $\partial_u\Omega \bigcap \partial_\sigma\Omega = \emptyset$, and constitutive Eqs. (3)–(5). Vectors $\boldsymbol{b}$, $\bar{\boldsymbol{t}}$, and $\bar{\boldsymbol{u}}$ are respectively applied body force, applied surface traction, and imposed displacement at time t.

Alternatively, the boundary value problem can be stated in variational form:

$$\operatorname*{stat}_{\dot{\boldsymbol{u}}} \inf_{\dot{d}} \int_\Omega \left[\dot{W}(\nabla^s \boldsymbol{u}, d) + \psi^*(\dot{d})\right] dV - \int_\Omega \boldsymbol{b} \cdot \dot{\boldsymbol{u}}\, dV - \int_{\partial\Omega_\sigma} \bar{\boldsymbol{t}} \cdot \dot{\boldsymbol{u}}\, dS \tag{11}$$