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Statistics of particle interactions in dense granular material under uniaxial compression

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Abstract

Stress evolution in a dense granular material is closely related to interactions of contacting particles. We investigate statistics related to particle interactions and the relationship between the averaged local relative motion and the macroscopic motion. The validity of the Voigt and Reuss assumptions is examined, and extensions to these assumptions are proposed. Effects of history in the dense granular material are investigated. Statistical samples used in this paper are obtained using three-dimensional numerical simulations of dense granular media under uniaxial cyclical compression. The results show that stresses arise mostly from normal forces between particles, and direct contributions from frictional tangential forces between particles are small. Tangential friction, however, significantly increases the particle contact time, and thus reduces the rate of contact breakage. The contact breakage rate is demonstrated to be a stress relaxation rate. Therefore, stress increases significantly with friction between particles as a result of prolonged relaxation time.

Keywords: Granular material; Dense packing; Powder compaction; Probability and statistics; Constitutive behavior

1. Introduction

Extensive research has been performed for dilute granular system with kinetic theories (Ben-Naim et al., 2005) and found successful application in many industrial applications,

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such as fluid beds and pneumatic transport. The assumption of instantaneous collisions between particles in dilute granular systems implies that only binary collisions are important, because a pair of particle completes their interaction before a third particle has a chance to collide with them. Multi-particle contacts in a dense granular system are consequence of finite particle contact time, which introduces great complexity compared to binary contacts. Multi-particle contacts limit the fluctuational motion of the particles. As a consequence, the streaming stress (the stress resulting from particle velocity fluctuations) which is important in dilute granular systems, is negligible compared to contact stresses in a dense granular system. The kinetic energy is also negligible compared with the potential energy in a dense system. Thus the extension of kinetic theory by Enskog (Chapman, 1970) to hard sphere dense fluid and Green–Kubo formulas (Lee, 2000; Goldhirsch and van Noije, 2000) cannot be applied directly to a closely packed dense granular medium consisting of deformable particles.

Slow dense granular flows are ubiquitous and have wide application (Jaeger, 2000; Hunt et al., 1999), for example, in powder metallurgy and compression molding composite engineering. In these industries powders or particles are pressed to form products. The deformation processes and the related constitutive relations are important for quality control. A common process of pulverizing ferro-alloys known as the cold-stream process has found increasing application for the production of very fine powder required for injection molding. This process can be modeled with dense granular material flow theory. The compaction of powders is a versatile fabrication route for monolithic metals (Fleck, 1995). Theoretical descriptions and modeling of such process remain one of the central challenges in the study of granular materials (Makse et al., 1999; Brujic et al., 2003; Corwin et al., 2005). In particular, characterizing the relaxation time of those composite materials which possess granular properties is a challenging problem in the modeling of the behavior of failure and forming processes of composite material. The present study examines how friction contributes to the total stress evolution based on the relation between the relaxation time and contact breakage rate (Zhang, 2005).

A practical granular system usually contains large amount of particles, with different sizes and shapes, interacting with each other. Because it is impossible to trace trajectories of individual particles, we employ a statistical approach. An important difference between a dilute granular flow and a dense granular flow resides in the relative importance of the finite particle contact time. Finite particle contact time in a granular system implies multi-particle interactions in the material. For a granular system with a finite particle contact time the evolution equation for the contact stress can be cast in a form similar to that used to describe many polymeric materials (Zhang and Rauenzahn, 1997, 2000; Zhang, 2005). Stress relaxation, as observed by Bowman and Soga (2003), is related to breakage of particle contacts. This is similar to the network theory (Bird et al., 1980) for polymers, in which the stress relaxation is related to breakage of polymer chains. Both the breakage of contacts in granular materials and the breakage of polymer chains are results of particle or atom motions. In the case of granular material, particle interactions are dissipative and the relative velocity between particles is proportional to the macroscopic strain rate, while in the case of polymers the relative motion between atoms is due to thermal motion. The additional relative velocity caused by an externally imposed strain rate in a polymer is negligible compared to the velocities of the thermal motion except under extreme conditions. As a consequence of these different origins of the relative velocities, the stress relaxation time for polymers depends strongly on temperatures but is almost independent