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Periodic truss structures

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ABSTRACT

Despite the recognition of the enormous potential of periodic trusses for use in a broad range of technologies, there are no widely-accepted descriptors of their structure. The terminology has been based loosely either on geometry of polyhedra or of point lattices: neither of which, on its own, has an appropriate structure to fully define periodic trusses. The present article lays out a system for classification of truss structure types. The system employs concepts from crystallography and geometry to describe nodal locations and connectivity of struts. Through a series of illustrative examples of progressively increasing complexity, a rational taxonomy of truss structure is developed. Its conceptual evolution begins with elementary cubic trusses, increasing in complexity with non-cubic and compound trusses as well as supertrusses, and, finally, with complex trusses. The conventions and terminology adopted to define truss structure yield concise yet unambiguous descriptions of structure types and of specific (finite) trusses. The utility of the taxonomy is demonstrated by bringing into alignment a disparate set of ad hoc and incomplete truss designations previously employed in a broad range of science and engineering fields. Additionally, the merits of a particular compound truss (comprising two interpenetrating elementary trusses) is shown to be superior to the octet truss for applications requiring high stiffness and elastic isotropy. By systematically stepping through and analyzing the finite number of structure types identified through the present classification system, optimal structures for prescribed mechanical and functional requirements are expected to be ascertained in an expeditious manner.

1. Introduction

Cellular structures and materials are ubiquitous in biological systems (Wainwright et al., 1982), structural engineering (Evans et al., 2001) and materials science (Gibson and Ashby, 1997). Broadly, they consist of periodic arrays of plate- or strut-like elements. They can be designed to most efficiently exploit the properties of the constituent elements and/or the intervening spaces in achieving functionality, *e.g.*, bearing loads, enabling fluid flow, facilitating heat transfer, altering optical transmission. They are generally superior to structures in which the elements are distributed in a non-periodic manner, *e.g.* stochastic foams (Evans et al., 2001). In some cases (*e.g.* photonic materials), periodicity is essential to achieving functionality. For load bearing applications, strut-like elements in particular are preferred: the load for initiating buckling of a slender strut being much higher than that of a comparable plate with the same mass.

Strut-based cellular structures and materials – hereafter collectively referred to as trusses – are under development for

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use in an incredibly broad range of technologies, including structural biomedical implants (Murr et al., 2010), aerospace and naval structures (Evans et al., 2001), cushioning and force protection systems (Wadley, 2000), thermal management (Wadley, 2000), actuated structures (Lucato et al., 2004; Hutchinson et al., 2003) and photonic materials (Bückmann et al., 2012; Bauer et al., 2014). Five main classes of fabrication routes have been employed.

(i) Investment casting has been used to make laboratory-scale Al-alloy truss structures (Deshpande et al., 2001; Chiras et al., 2002; Wallach and Gibson, 2001). Investment casting is generally the most expensive and least amenable to large-scale production relative to other fabrication routes.

(ii) Fabrication schemes based on conventional machining, bending, assembly and brazing of sheet materials have been devised to make metallic trusses (Wadley, 2000; Rathbun et al., 2004). In one version, diamond-shaped holes are punched or laser-machined into thin steel sheet, leaving an X-pattern of narrow struts. The sheet is then bent along lines of nodes to produce one layer of the targeted truss (Wadley et al., 2003).

(iii) Metallic trusses can also be made by weaving wires into the desired structure and subsequently brazing the wires together (Wadley, 2000; Kang, 2015). One of the drawbacks is that the weaving operations yield wavy or kinked strut segments between nodes. Moreover, since the nodes are formed by brazing of contacting wires, the integrity of these nodes is likely to be strength-limiting.

(iv) Additive manufacturing offers the widest range of topology options. Some of the most notable developments in recent years have been in Ti-alloy trusses, produced by selective electron beam melting (EBM) of fine alloy powders, for biomedical implants (Murr et al., 2010, 2011; Cheng et al., 2012; Li et al., 2014). In another arena, direct laser writing by optical lithography has been used to fabricate polymer truss structures with extremely fine-scale (sub-micrometer) features, for potential use in photonic applications (Bückmann et al., 2012; Bauer et al., 2014).

(v) Self-propagating photocuring (*SPPC*) of photosensitive polymers has found utility in rapid fabrication of polymer trusses for use in impact mitigation and cushioning systems (Jacobsen et al., 2007a, 2007b, 2008). The main advantage of this process is the short time needed for polymerization (typically less than a minute). One significant limitation is the narrow range of materials that exhibit the requisite physical and chemical properties for SPPC as well as the mechanical properties to produce useful truss structures. It is also restricted to topologies in which all struts intersect one of the external faces. That is, it is inherently a “line-of-sight” curing method.

Despite the broad recognition of the potential of periodic trusses for use in many diverse fields of technology, there are no widely-accepted descriptors of their structure. In the numerous articles on this topic that have appeared in the past two decades, the terminology has been based loosely on descriptions of various polyhedra, but often without explicit connections between truss structure and specific characteristics of the reference polyhedron.

For example, trusses designated as pyramidal are conceptually constructed by placing struts along the four edges of a regular square pyramid at which the triangular faces intersect, but not along the edges of the square base (Evans et al., 2001; Wadley, 2000). Similarly, tetrahedral trusses are formed by placing struts along three non-coplanar edges of a tetrahedron, but not on the other three edges (Wadley, 2000; Rathbun et al., 2004). In other cases, truss structures are constructed by placing struts normal t*o* and at the center of each face of the reference polyhedron (*not* along the edges), *e.g.* the truncated octahedral truss (Gurtner and Durand, 2014; Weaire, 1996). Elsewhere, truss structures have been described as being “tetrahedral with three-fold symmetry” or “tetrahedral with six-fold symmetry”, without explicit designations of strut locations (Jacobsen et al., 2008).

In some instances, new words have been devised to describe truss structure. The octet truss, for example, derives from a combination of octahedral and tetrahedral. Here struts are placed along all edges of a series of regular octahedra and tetrahedral arranged to fill three-dimensional space (Deshpande et al., 2001). Other truss structures have been described loosely as “fully triangulated”, “bulk cross” (Kang, 2015), “cross I symmetric”, “G6”, “G7”, “dode-thin”, and “hatched” (Murr et al., 2010, 2011; Cheng et al., 2012). These and the preceding designations are re-visited in a later section of this article.

In addition to the vagaries introduced by using polyhedra as the basis of truss designations, the terminology fails to recognize the fundamentally different nature of polyhedra and of trusses. A polyhedron is a three-dimensional solid whose outer boundaries are defined by plane polygons such that the edge of each polygon belongs to one other polygon. A truss, on the other hand, consists of a set of points (or nodal locations) and a set of lines (or struts) joining certain points. Solid geometry alone lacks the structure needed to completely and unambiguously describe truss structure.

Descriptions of trusses have also frequently invoked terms derived from the field of crystallography. Examples include “body centered cubic” and “diamond”. Indeed, the association between nodal positions of trusses and space lattices in crystallography has led to the characterization of trusses as lattice materials, lattice structures or simply lattices. In addition to the unfortunate conflict with the definitions of lattices in the context of crystallography, the terminology (again) fails to recognize the fundamental differences between space lattices and truss structure: A space lattice defines only an array of regularly-spaced points and provides no information about the connectivity of those points (*i.e.* topology). Therefore, crystallography alone (like solid geometry) lacks the structure needed to describe truss structure.