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Metamaterial properties of periodic laminates

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ABSTRACT

In this paper we show that a 1-D phononic crystal (laminate) can exhibit metamaterial wave phenomena which are traditionally associated with 2- and 3-D crystals. Moreover, due to the absence of a length scale in 2 of its dimensions, it can outperform higher dimensional crystals on some measures. This includes allowing only negative refraction over large frequency ranges and serving as a near-omnidirectional high-pass filter up to a large frequency value. First we provide a theoretical discussion on the salient characteristics of the dispersion relation of a laminate and formulate the solution of an interface problem by the application of the normal mode decomposition technique. We present a methodology with which to induce a pure negative refraction in the laminate. As a corollary to our approach of negative refraction, we show how the laminate can be used to steer beams over large angles for small changes in the incident angles (beam steering). Furthermore, we clarify how the transmitted modes in the laminate can be switched on and off by varying the angle of the incident wave by a small amount. Finally, we show that the laminate can be used as a remarkably efficient high-pass frequency filter. An appropriately designed laminate will reflect all plane waves from quasi-static to a large frequency, incident at it from all angles except for a small set of near-normal incidences. This will be true even if the homogeneous medium is impedance matched with the laminate. Due to the similarities between SH waves and electromagnetic (EM) waves it is expected that some or all of these results may also apply to EM waves in a layered periodic dielectric.

1. Introduction

The observation that periodic structures can significantly affect wave propagation has inspired a large body of scientific research in the areas of both photonics and phononics over the last two decades. The excitement stems from the applications which have been proposed of these structures. Some of these include negative refraction, beam splitting, beam steering, and frequency filtering. Traditionally, the focus has been on 2-D and 3-D periodic structures with 1-D structures (laminates) serving merely as a stepping stone to more interesting problems in higher dimensions. What the laminates gained in the ease of their assembly, they lost in the apparent lack of richness in their wave physics.

The body of literature is large and here we direct the reader to some review papers on these topics (Pennec et al., 2010; Lee et al., 2012; Hussein et al., 2014; Srivastava, 2015). This paper considers a specific problem within this field: oblique incidence of shear horizontal (SH) waves at an interface between a homogeneous material and a layered periodic composite. As such, the problem is similar to the propagation of EM waves in a layered periodic dielectric. We assume that the layer

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interfaces are in the x_2–x_3 plane. The analysis constitutes determining the dispersion relation of the laminate for a wave-vector in the x_1–x_2 plane. Although a host of numerical techniques could be used, this problem is normally simple enough to admit an exact solution. The approach is well established in the form of the transfer matrix method (TMM). TMM can tackle both normal incidence (Lin and McDaniel, 1969; Faulkner and Hong, 1985; Kutsenko et al., 2015) and oblique incidence (Lekner, 1994) with the former being a special case of the latter. In this paper we use the approach used by Lekner (1994) for EM waves which was subsequently adapted by Willis recently for SH waves (Willis, 2015) in solids. The dispersion relation of the laminate in the context of mechanics has been studied by several authors over the last five decades (Sun et al., 1968; Nemat-Nasser, 1972; Nemat-Nasser et al., 1975; Hegemier and Nayfeh, 1973; Hegemier and Bache, 1973). For the EM case the contributions go back even further with some of the first studies from Lord Rayleigh now more than a hundred years old (Rayleigh, 1887, 1917) (see also Joannopoulos et al., 2011). That laminates exhibiting dispersion and bandgaps have, therefore, been common knowledge for a considerable time now.

With the advent of photonic/phononic crystals and metamaterials, interest subsequently increased in the manifestation of exotic wave phenomena including, but not limited to, negative refraction. The implicit assumption here is the existence of an interface of these crystals with another medium. The interface problem with a homogeneous material can be comprehensively studied through a combination of the TMM and normal mode decomposition. TMM provides information about not just the propagating modes but also the evanescent modes. Normal mode decomposition, on the other hand, tells us exactly how the energy is partitioned at the interface among the various available modes. With this, the study of such anti-plane shear waves (SH in solids or EM waves) can be broadly divided into two categories on the basis of the location of the interface between the homogeneous material and the laminate: (a) when the interface lies in the x_2–x_3 plane and (b) when the interface lies in the x_1–x_3 plane. More general oblique interfaces are also possible but they are not considered here.

Of the two problem sets the former is the more conventional one (Boudouti and Djafari-Rouhani, 2013). Within this, perhaps the most important (and most relevant here) observation, beyond the simple bandstructure effect, is the realization that laminates in this configuration that can be used as omnidirectional reflectors (Winn et al., 1998; Fink et al., 1998; Bousfia et al., 2001; Manzanares-Martínez et al., 2004). The idea is to choose a combination of the frequency range, homogeneous material and the laminate such that the 2-component of the wave vector in the homogeneous material can never couple to a propagating mode in the laminate. In this paper we observe that the laminate can be used as a highpass frequency filter. Such a filter will reflect all the waves from a quasi-static frequency to an arbitrarily large frequency value and for all angles except for an arbitrarily small range near normal incidence.

The latter set of problems have received attention only recently through papers by Willis (2015) and Nemat-Nasser (2015a, 2015b). It has been suggested that in this configuration it is possible to achieve negative refraction in a laminate – a phenomenon traditionally associated with more complex 2- and 3-D crystals. Willis presented TMM+normal mode decomposition calculations and showed that at a chosen frequency the transmitted signal consisted of both positively and negatively refracted signals (beam splitting). In this configuration we show that it is possible to achieve pure negative refraction in the laminate (no beam splitting with a positive refraction). This is achieved by coupling the wave in the homogeneous medium with the laminate modes *not in the first* but in the second Brillouin zone at frequencies where only one propagating mode exists. This is not possible when the interface is in the x_2–x_3 plane or in 2- and 3-D composites generally. Negative refraction achieved in this fashion is persistent over large ranges of angles of incidence and frequency. Second, we show that there exists frequencies where the positive and negative refraction angles are very large and that the beam can be made to traverse these large angles with only a small change in the incidence angle (beam steering). Finally, we show that the transmitted beams can be switched on and off through small changes in the angles of incidence.

In the following sections we present the relevant equations following Lekner (1994) and Willis (2015) (Section 2). We complement these equations with theoretical discussions which illuminate certain essential characteristics of the dispersion relation. Poynting vector calculations are presented in Section 3 as the unambiguous method of determining refractive possibilities of various modes. This treatment is different but equivalent to the Equi-Frequency Surface (EFS) based arguments (Notomi, 2000). In Section 4 we summarize our observations for the case when the interface is in the x_1–x_3 plane (negative refraction, beam steering, and mode switching). TMM+normal mode decomposition equations are presented and energy based checks on the validity of the calculations are formulated. In Section 4 we summarize our observation for the case when the interface is in the x_2–x_3 plane (high-pass filtering).

2. Relevant equations

Following Willis (2015) we define our laminate as a periodically layered structure in the x_1 direction with the layer interfaces in the x_2–x_3 plane and infinite in this plane. In the direction of periodicity the laminated composite is characterized by a unit cell Ω of length h ($0 \le x_1 \le h$). For our purposes the unit cell is composed of 2 material layers with shear moduli μ_1, μ_2, density ρ_1, ρ_2, and thicknesses h_1,h_2 respectively. The case of n homogeneous material layers or layers with spatially changing material properties is not substantively different. All that is required is that the material properties be periodic with the unit cell so as to give the composite its phononic character.

If anti-plane shear waves are propagating in the laminate then the only nonzero component of displacement is taken to be u_3 which has the functional form $u_3(x_1, x_2, t)$. Within the ith layer ($i=1,2$) it satisfies the following equation of motion: