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A nonlinear mechanics model of bio-inspired hierarchical lattice materials consisting of horseshoe microstructures

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ABSTRACT

Development of advanced synthetic materials that can mimic the mechanical properties of non-mineralized soft biological materials has important implications in a wide range of technologies. Hierarchical lattice materials constructed with horseshoe microstructures belong to this class of bio-inspired synthetic materials, where the mechanical responses can be tailored to match the nonlinear J-shaped stress–strain curves of human skins. The underlying relations between the J-shaped stress–strain curves and their microstructure geometry are essential in designing such systems for targeted applications. Here, a theoretical model of this type of hierarchical lattice material is developed by combining a finite deformation constitutive relation of the building block (i.e., horseshoe microstructure), with the analyses of equilibrium and deformation compatibility in the periodical lattices. The nonlinear J-shaped stress–strain curves and Poisson ratios predicted by this model agree very well with results of finite element analyses (FEA) and experiment. Based on this model, analytic solutions were obtained for some key mechanical quantities, e.g., elastic modulus, Poisson ratio, peak modulus, and critical strain around which the tangent modulus increases rapidly. A negative Poisson effect is revealed in the hierarchical lattice with triangular topology, as opposed to a positive Poisson effect in hierarchical lattices with Kagome and honeycomb topologies. The lattice topology is also found to have a strong influence on the stress–strain curve. For the three isotropic lattice topologies (triangular, Kagome and honeycomb), the hierarchical triangular lattice material renders the sharpest transition in the stress–strain curve and relative high stretchability, given the same porosity and arc angle of horseshoe microstructure. Furthermore, a demonstrative example illustrates the utility of the developed model in the rapid optimization of hierarchical lattice materials for reproducing the desired stress–strain curves of human skins. This study provides theoretical guidelines for future designs of soft bio-mimetic materials with hierarchical lattice constructions.

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1. Introduction

Research over the last decade has yielded rapid and substantial advancements in the field of materials science that draws inspiration from the nature, as an important approach to the design and synthesis of new materials (Aizenberg, 2005; Capadona et al., 2008; Cranford et al., 2012; Kim et al., 2013; Ma et al., 2013; Morin, 2012; Ortiz and Boyce, 2008; Pokroy et al., 2009; Sanchez et al., 2005; Wegst et al., 2015; Wong, 2011). Among many examples, bio-inspired structural materials are of growing interest, due to utility of the types of complex and hierarchical micro/nano-structures that are found in most biological systems (Meyers et al., 2013; Ortiz and Boyce, 2008; Wegst et al., 2015). The mechanical and functional performances are attractive for applications in a wide range of engineered systems.

Two broad classes of structural materials can be found in biology (Meyers et al., 2013): (1) mineralized hard materials, mainly in the form of hierarchically assembled composites that combine minerals (e.g., calcium carbonate and amorphous silica) with organic polymer additives (e.g., collagen); and (2) non-mineralized soft materials, typically constructed with wavy, fibrous constituents (e.g., collagen, keratin and elastin) that are embedded in extracellular matrices. Representative examples of the former type include bone, seashell and teeth, which exhibit remarkable combinations of high stiffness and toughness. The underlying mechanisms of their extraordinary mechanical properties associate their resistance to fracture with hierarchical constructions of microstructures (Evans et al., 2001b; Gao et al., 2003; Jackson et al., 1988; Launey et al., 2010; Lin and Meyers, 2009; Schaffer et al., 1997; Song and Bai, 2001). In particular, the 'brick-and-mortar' arrangement of organic constituents and minerals enhances the fracture toughness, partially due to the deflection of cracks around the 'bricks' instead of through them (Launey et al., 2010). A number of synthetic materials with similar microstructure constructions were fabricated and characterized (Bonderer et al., 2008; Bouville, 2014; Mayer, 2005; Munch et al., 2008; Tang et al., 2003; Weiner and Addadi, 1997). Furthermore, both theoretical and computational models were developed to study the various mechanical properties (e.g., elastic modulus, ultimate strength, buckling resistance), to provide important guidelines for optimal design (Buehler et al., 2006; Ji, 2008; Ji and Gao, 2004, 2006, 2010; Ji et al., 2004; Kauffmann et al., 2005; Yao and Gao, 2007; Zhang et al., 2010, 2011). By contrast, non-mineralized soft biological materials refer to biopolymers such as collagen and viscid spider silk, which possess simultaneously low elastic moduli, large levels of stretchability, and relatively high tensile strengths. Recent studies show that an unconventional J-shaped stress–strain curve, induced by molecular uncoiling and unkinking under low stress, can yield superior mechanical properties (Fratzl et al., 1998; Gautieri et al., 2011; Keten et al., 2010; Komatsu, 2010; Meyers et al., 2013; Miserez et al., 2009; Provenzano et al., 2002; Simmons et al., 1996). Despite promising applications in tissue engineering and biomedical devices, the development of soft synthetic materials with matching mechanical properties has received far less attention (Hong, 2011; Jang et al., 2015; Naik et al., 2014) compared to that of mineralized biological materials, in part due to the complex, irregularly distributed microstructures.

Recently, Jang et al. (2015) introduced a class of soft network composite material that embeds an ultralow-modulus ($\sim$3 kPa) matrix with an open, stretchable network as a structural reinforcement. The studied network consists of a hierarchical lattice pattern that combines two-dimensional (2D) lattice topologies (e.g., triangular, Kagome and honeycomb) of cellular materials (Chen et al., 1999; Deshpande et al., 2001; Evans et al., 2001a; Fleck and Qiu, 2007; Hutchinson and Fleck, 2006; Kang et al., 2014, 2013; Lu and Chen, 1999; Zhang et al., 2008) with stretchable horseshoe/serpentine microstructures (Hsu et al., 2009; Kim et al., 2011, 2008; Widlund et al., 2014). Preliminary finite element analyses (FEA) and experimental measurements (Jang et al., 2015) demonstrate that such hierarchical lattice materials can be tailored to match the nonlinear J-shaped stress–strain curves of human skin, thereby offering great promise for applications in tissue engineering (Naik et al., 2014; Yannas and Burke, 1980). Such J-shaped stress–strain curves combine soft, compliant mechanics and large levels of stretchability, with a huge modulus enhancement at large strain that offers a relatively high mechanical strength simultaneously. Integrating all of these mechanical attributes in a single system is very attractive for achieving a mechanically robust form of stretchable electronics (Jang et al., 2014) that could improve the survivability substantially in bio-integrated applications (Kim et al., 2011; Rogers et al., 2010; Xu et al., 2014; Yao and Zhu, 2014; Zhang et al., 2014). The underlying relations between the J-shaped stress–strain curves and the microstructure geometric parameters of hierarchical lattice materials require, however, a relevant mechanics theory, as the basis of a design approach for practical applications. Optimization of hierarchical lattice materials for desired nonlinear mechanical response is prohibitively time-consuming based on FEA. In this study, a theoretical model of hierarchical lattice materials is developed to study the deformation mechanisms and to predict the J-shaped stress–strain curves. Quantitative comparisons with FEA and experimental results illustrate its validity. Based on this model, analytic solutions were obtained for some key mechanical quantities (e.g., elastic modulus, peak modulus, Poisson ratio and critical strain around which the tangent modulus increases rapidly). Both negative and positive Poisson ratios were found in this class of hierarchical lattice materials, which show nonlinear and anisotropic characteristics at large levels of stretching. Furthermore, a demonstrative example shows that the developed model can be employed to enable rapid optimization of microstructure geometry for matching precisely the stress–strain curves of human skins.

The paper is outlined as follows. Section 2 illustrates the design concept and geometry of the hierarchical lattice materials. Section 3 describes a finite deformation model of the building block, i.e., a filamentary wire in the horseshoe pattern. In Section 4, this model was combined with the analyses of equilibrium and deformation compatibility in the periodical lattices, to formulate a theoretical model for the hierarchical lattice materials. Section 5 presents the analysis of key mechanical properties as well as the effect of lattice topology, using the developed theoretical model and FEA. Section 6