Journal of the Mechanics and Physics of Solids 90 (2016) 215–241

Contents lists available at ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Granular vortices: Identification, characterization and conditions for the localization of deformation

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A R T I C L E I N F O

Article history:
Received 23 June 2014
Received in revised form
14 September 2015
Accepted 26 February 2016
Available online 9 March 2016

Keywords:
Granular material
Shear bands
Force chain buckling
Vortices
Continuum micromechanics

A B S T R A C T

We relate the micromechanics of vortex evolution to that of force chain buckling and, on this basis, formulate the conditions for strain localization in a continuum model of dense granular media. Using the traditional bifurcation analysis of shear bands, we show that kinematic vortex fields are in fact solutions to the boundary value problem satisfying null boundary conditions. To establish an empirical basis for our study, we first develop a method to identify the location of the core and boundary of each vortex from a given displacement field in two dimensions. We then employ this method to characterize the residual deformation field (i.e., the deviation of particle motions from the continuum deformation) in a physical experiment and a discrete element simulation of dense granular samples submitted to biaxial compression. Vortices in the failure regime are essentially confined to the shear band. Primary vortices, the clear majority, rotate in the same direction as the shear band; secondary vortices, the so-called wakes, rotate in the opposite direction. Primary vortices align in spatial succession along the central axis of the band; wakes form next to the band boundaries, in between and beside two adjacent primary vortices. Force chain buckling, the governing mechanism for shear bands, is responsible for vortex formation in the failure regime. Vortex dynamics are consistent with stick-slip dynamics. From quiescent conditions of jamming or stick, vortical motions arise from force chain buckling and associated relative particle rotations and sliding; these in turn precipitate intermittent periods of unjamming or slip, evident in the attendant drops in stress ratio and bursts in both kinetic energy and local nonaffine deformation. A kinematic vortex field inside shear bands is proposed that is consistent with the equations of continuum mechanics and the underlying instability of force chain buckling: such a field is periodic with a repeating unit cell comprising a primary vortex at the center of the band, with two trailing wakes close next to the band boundaries.

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http://dx.doi.org/10.1016/j.jmps.2016.02.032
0022-5096/

1. Introduction

A flow pattern characteristic of failure in rate-independent materials is the localization of strain into the so-called shear bands — the conditions for which were first proposed forty years ago by Rudnicki and Rice (1975). Their theory was set forth in the framework of continuum mechanics: shear band formation is treated as a bifurcation in the solution path of the boundary value problem for a continuum flow field, such that a pattern of localized deformation is permitted by the constitutive law as an additional solution to the "trivial" uniform solution. Despite its success, two key challenges remain in the mechanics and physics of shear bands in dense granular media. First, the full details of grain motions that precede and emerge in fully developed shear bands are only just beginning to come to light (Rechenmacher et al., 2011; Andò et al., 2012; Peters and Walizer, 2013; Tordesillas et al., 2014). Second, the mathematical route to bridge the dichotomy between the micromechanical description of grain-scale instabilities central to shear band formation versus the phenomenological description of continuum theory, where the practical analysis of geotechnical structures and geomaterial phenomena lie, remains unclear (Tordesillas et al., 2014). Advances in noninvasive imaging techniques (Oda and Kazama, 1998; Desrues and Viggiani, 2004; Majmudar and Behringer, 2005) and numerical discrete element methods (DEM) (Cundall and Strack, 1979; Iwashita and Oda, 1998; Oda and Iwashita, 2000) have led to extensions of the Rudnicki and Rice theory within the framework of micropolar continuum mechanics (Muhlhaus and Vardoulakis, 1987; Sulem and Vardoulakis, 1995; Tordesillas et al., 2004), yet fundamental questions remain unanswered. Foremost of these are the significance of vortex patterns in grain motions that prevalently emerge inside shear bands and the mechanisms behind the evolution of these vortices.

The connection between vortices and shear bands has attracted rife speculation (e.g. Williams and Rege, 1997; Kuhn, 1999; Alonso-Marroquín et al., 2006; Tordesillas et al., 2008a; Rechenmacher et al., 2010, 2011; Liu et al., 2012; Peters and Walizer, 2013; Kozicki et al., 2013; Tordesillas et al., 2014). Recently, this connection was clarified from two fronts. First, Peters and Walizer (2013) observed that a slip field can be expressed as the sum of a uniform (affine) deformation field and a vortex. This is an obvious result of a simple shear field – which characterizes the deformation of the shear band – being a linear combination of a pure shear deformation and a rigid-body rotation. Though the analysis by Peters and Walizer (2013) was directed at the pre-failure and not post-localized failure regime, it holds important implications for shear band analysis. They observed vortices early in the loading history, which then implied a slip field in the area where the shear band would ultimately form. Their findings were consistent with those from analysis of high-resolution microCT data on individual grain kinematics, which demonstrated that the ultimate pattern of failure is encoded in the grains' motions from the beginning of loading in a triaxial compression of Hostun sand (Tordesillas et al., 2013) and, more recently, of Ottawa and Caicos Ooid sand, as well as in a DEM simulation of simple shear (Tordesillas et al., 2015). Second, important structural building blocks of 3-cycles (i.e., three particles in mutual contact, known to provide truss-like supports to column-like force chains), exhibit similar vortex patterns early in the loading history (Tordesillas et al., 2014). A multi-scale spatio-temporal analysis of these 3-cycle structures in the pre-failure regime revealed a preferential and progressive degradation of the most stable 3-cycles (the so-called persistent 3-cycles) – *where the shear band ultimately forms*. In fact, the location and strain state at which these structures became essentially depleted marked the full development of the shear band in space and time, and explained why force chains are prone to buckling *inside* the band from thereon. Use of the generalized Ripley K-function (Cressie, 1993), to quantify the existence of spatial attraction (i.e., significant statistical evidence two distinct events that frequently occur in close proximity of each other), uncovered a pattern in the dynamics of 3-cycles and force chains that is unique to the failure regime (Tordesillas et al., 2014). Once the persistent shear band is fully formed, 3-cycle "births" and "deaths" exhibit spatial attraction in the band region, as do 3-cycle deaths and buckling force chains, to within a length scale of 7–10 grain diameters, consistent with the band thickness. Energy released from these failure events then drive the vortical nonaffine motions, which are confined to the band and intensify during periods of unjamming.

These recent developments underscore the need to characterize comprehensively the relationship between vortex evolution inside shear bands and the mechanism of force chain buckling, long implicated as the root cause of shear banding (Oda and Kazama, 1998; Oda and Iwashita, 2000; Iwashita and Oda, 2000; Tordesillas et al., 2008a, 2009b; Rechenmacher et al., 2010, 2011; Liu et al., 2012; Tordesillas et al., 2014). There is compelling ground for these mechanisms to be related, after all both concentrate in shear bands in the failure regime. Moreover, micromechanical studies have implicated force chain evolution in various failure phenomena observed across the mesoscopic and macroscopic scales including: *noncoaxiality of stress and strain rate* (Collins and Muhunthan, 2003; Thornton and Zhang, 2006; Tordesillas et al., 2009a), *strain-softening under dilatation* (Oda and Kazama, 1998; Rechenmacher, 2006; Tordesillas et al., 2011b), *stick-slip dynamics* (Griffa et al., 2011), and *spatial interlacing of high and low porosity regions inside shear bands* (Oda and Kazama, 1998; Oda and Iwashita, 2000; Iwashita and Oda, 2000; Rechenmacher, 2006; Tordesillas et al., 2008a).

Here we propose that the seeds of the unstable behavior assumed in the bifurcation analysis of Rudnicki and Rice (1975) can in fact be identified from the underlying mechanism that drives the formation of vortex patterns in grain motions and, ultimately, shear bands. We choose the biaxial compression test, the principal advantage of which lies in the boundary conditions: these give rise to a known affine deformation field that serves as a backdrop on which to explore the nonaffine vortex evolution in a granular material as it transitions into failure by strain localization (Williams and Rege, 1997).

We establish the conditions for localization of deformation with respect to a kinematic vortex field inside shear bands that is consistent with the underlying particle scale instability driving vortex and shear band evolution, i.e., force chain buckling. Our analysis proceeds in three phases. In the first phase (Section 2), we develop an objective method by which to identify vortices from a given displacement field in two dimensions. Armed with this vortex detection method, we next