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An alternative approach to microplane theory

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ABSTRACT

In this paper an alternative approach to the well-known microplane theory is developed. This approach mainly consists of two parts, i.e. the mathematical representation based on the improved representation theorem on stiffness and the equivalent thermodynamical description within the framework of irreversible thermodynamics. On one hand, the material stiffness is represented in the form of irreducible decomposition which can be sufficiently determined by the orientation distribution functions for the macroscopic bulk and shear moduli (or those for the corresponding macroscopic damages). The introduced macroscopic orientation distribution functions are then expanded into the converged Fourier series and approximated by the second- or fourth-order macroscopic damage variables which are defined as the fabric tensorial functions of the microscopic (microplane) damage variables. The combination of the improved representation theorem on stiffness with the proposed macroscopic and microscopic damage variables yields the general forms of the microplane models with bulk-shear split and with volumetric–deviatoric–tangential split. On the other hand, the macroscopic Helmholtz free energy potentials are defined by introducing the damage effect tensors in terms of the macroscopic damage variables. The integral relation between the microscopic and macroscopic Helmholtz free energy potentials as well as the kinematic constraint is derived. Within the framework of irreversible thermodynamics, the consistent microscopic and macroscopic damage evolution laws are established. Moreover, the other concepts of damage mechanics such as the conjugated damage forces, the damage dissipations and so on, are also investigated on both the microscopic and macroscopic levels.

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1. Introduction

Materials generally contain a multitude of defects in the form of microvoids. Upon the initial state it is reasonable to assume that the distribution of microvoids is statistically homogeneous in the initially isotropic materials. During the loading history, these microvoids as well as the newly initiated ones may evolve into the so-called microcracks that undergo irreversible growth mainly "in the direction perpendicular to the maximum tensile strain or stress" (Krajcinovic and Fonseka, 1981). The microcracks evolution inevitably leads to the microstructural changes from the initial isotropy to anisotropy (i.e. the so-called damage induced anisotropy). The inelastic isotropic material behavior is nowadays well-understood in either the plasticity (Chen, 1994) or damage mechanics (Krajcinovic, 2003). On the contrary, although large quantities of relevant researches have been carried on, the modeling of damage induced anisotropy is far more difficult and poses great challenges to both the scientific and engineering communities.

Generally speaking, damage induced anisotropy is the macroscopic characterization of the heterogeneous microstructure changes. Regarding to this fact, the micromechanics based approaches (Budiansky, 1965; Hill, 1965; Budiansky and O'Connell, 1976; Horii and Nemat-Nasser, 1983; Budiansky, 1983) have some intrinsical advantages

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especially attribute to the great progresses recently made in the micromechanical damage mechanics (Nemat-Nasser and Hori, 1999; Krajcinovic, 2003). In these methods, the influences of a single microvoid or microcrack are studied based on the elastic-fracture mechanics and the overall properties of the heterogeneous material are then obtained by appropriate microscopic–macroscopic (micro–macro) homogenization scheme. However, due to the great difficulties in considering the strong microcracks interactions (Kachanov, 1987, 1992, 1993) and in identifying the actual microcracks distributions (Lubarda and Krajcinovic, 1993) the micromechanics based approaches remain unpractical to model the damage induced anisotropy, at least in the near future. Comparatively, due to the conceptual simplicity the macroscopic or continuum damage mechanics (CDM) gains great popularity in the inelastic constitutive modeling; see (Lemaitre and Chaboche, 1990; Krajcinovic, 2003) and the references therein. In the CDM model, the damage variable is introduced to macroscopically represent the underlying microstructural process within the framework of irreversible thermodynamics. As concluded by Kanatani (1984), only second or higher even-order damage tensor is appropriate to describe the damage induced anisotropy. Nevertheless, under the general 3-D loading history it is rather difficult to postulate an appropriate evolution law for the tensorial damage variable, even for the relative simpler second-order damage tensor (Carol et al., 2001). Moreover, upon cyclic loading the microcracks closure–reopening (MCR) effects make the problem even more complex (Carol and Willam, 1996; Wu and Li, 2007).

It is therefore desirable to combine the advantages of both the micromechanics and CDM based approaches. Such a general concept for the material modeling is to develop constitutive law on characteristic planes. The first application of this approach might be traced back to the classical Tresca and Mohr-Coulomb plasticity (Mohr, 1900). Based on this concept, the "slip theory of plasticity" was established for crystalline materials (Taylor, 1938; Batdorf and Budiansky, 1949) and was later adapted to the so-called multilaminate model for fractured rocks and soils (Zienkiewicz and Pande, 1977; Pande and Sharma, 1983). These models generally yield stable predictions only in the hardening regions due to the assumed static constraint (Bažant et al., 1983). Obviously, this deficiency poses great restrictions when applied to the quasi-brittle materials like concrete. During the last two decades, Bažant and coworkers turned to the kinematic constraint and developed the well-known microplane theory for the quasi-brittle materials (Bažant et al., 1983; Bažant and Gambarova, 1984; Bažant and Oh, 1985; Bažant and Prat, 1988; Carol et al., 1991, 1992; Bažant et al., 1996; Carol and Bazant, 1997; Bažant et al., 2000). On one hand, the microplane theory concerns with the microscopic analysis of the material behavior on a single microplane and then the micro–macro homogenization is established to obtain the overall macroscopic properties. On the other hand, it was found that the microplane theory is in close relation to CDM concepts (Carol et al., 1991; Carol and Bazant, 1997; Kuhl et al., 2001). Therefore, the microplane theory can be regarded as a combined micro–macro damage model characterized by the following two distinct advantages: (1) the damage induced anisotropy as well as the MCR effects can be intrinsically considered, and (2) the complex 2-D or 3-D inelastic material modeling can be established based on the relatively simpler 1-D modeling.

There indeed exist some versions of microplane theory adopting the static constraint (Carol and Prat, 1990; Carol et al., 1992; Carol and Prat, 1995) or mixed kinematic-static constraint (Bažant and Caner, 2005) which are, however, rarely used probably due to the troublesome numerical implementation. In this paper only the microplane models with kinematic constraint are to be considered. The kinematic constraint means that the microscopic strains on a specific microplane are equal to the projections of the macroscopic strain tensor. The microplane kinematics and kinetics are generally characterized by the normal and tangential strains and the corresponding stress components (i.e. N–T split). The microplane model with N–T split (Bažant et al., 1983; Bažant and Gambarova, 1984; Bažant and Oh, 1985) worked well for tensile cracking but was incapable of modeling the inelastic behavior under compression and shear. Therefore, nearly all the subsequent microplane models (Bažant and Prat, 1988; Carol et al., 1991, 1992, 2001, 2004; Bažant et al., 1996, 2000; Ožbolt et al., 2001; Kuhl et al., 2001) adopted the so-called volumetric–deviatoric–tangential split (namely, V–D–T split), i.e. the microplane normal strain and stress are further split into the volumetric and deviatoric components. Recently, a restricted microplane model with bulk-shear split (i.e. B–S split) which can be viewed as a special case of the more general V–D–T split based formulation was proposed (Leukart and Ramm, 2002, 2003, 2006). In this split methodology, the macroscopic strain is decomposed into its bulk and shear components which are then projected onto the specific microplane to obtain the microscopic stress components.

The kernel of microplane theory is that the 2-D or 3-D constitutive laws can be obtained by integrating the behavior of a generic microplane predefined over all possible spatial orientations. To this end, a micro–macro homogenization scheme is required. The best choice would be the *strong* micro–macro force equilibrium relation which means double kinematic-static constraint. Nevertheless, this double constraint does not hold for the general microscopic material laws and only the approximation in a *weak* sense can be expected. Since the principle of virtual work (PVW) based micro–macro homogenization can be viewed as the least-square optimization of the microplane stresses determined from the static constraint (Carol and Bazant, 1997), it seems natural that the PVW based formulation was adopted in the early microplane models (Bažant and Oh, 1985; Bažant and Prat, 1988; Carol et al., 1992; Bažant et al., 1996). During this stage, the microplane model with N–T split is a special case of the one with V–D–T split. Though mainly motivated by the phenomenological observation, the PVW based microplane theory yields excellent data fitting in the numerical modeling. However, Carol et al. (2001) pointed out that such derived microplane model with V–D–T split violates the second principle of thermodynamics due to the non-symmetric material stiffness. More specifically, the microplane deviatoric stress is not consistently defined, leading to spurious energy

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