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Rolling contact of a rigid sphere/sliding of a spherical indenter upon a viscoelastic half-space containing an ellipsoidal inhomogeneity

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ABSTRACT

In this paper, the frictionless rolling contact problem between a rigid sphere and a viscoelastic half-space containing one elastic inhomogeneity is solved. The problem is equivalent to the frictionless sliding of a spherical tip over a viscoelastic body. The inhomogeneity may be of spherical or ellipsoidal shape, the later being of any orientation relatively to the contact surface. The model presented here is three dimensional and based on semi-analytical methods. In order to take into account the viscoelastic aspect of the problem, contact equations are discretized in the spatial and temporal dimensions. The frictionless rolling of the sphere, assumed rigid here for the sake of simplicity, is taken into account by translating the subsurface viscoelastic fields related to the contact problem. Eshelby's formalism is applied at each step of the temporal discretization to account for the effect of the inhomogeneity on the contact pressure distribution, subsurface stresses, rolling friction and the resulting torque. A Conjugate Gradient Method and the Fast Fourier Transforms are used to reduce the computation cost. The model is validated by a finite element model of a rigid sphere rolling upon a homogeneous vciscoelastic half-space, as well as through comparison with reference solutions from the literature. A parametric analysis of the effect of elastic properties and geometrical features of the inhomogeneity is performed. Transient and steady-state solutions are obtained. Numerical results about the contact pressure distribution, the deformed surface geometry, the apparent friction coefficient as well as subsurface stresses are presented, with or without heterogeneous inclusion.

1. Introduction

Polymer based materials are extensively used in several engineering domains due to their numerous advantages such as low cost of raw materials and less complex manufacturing, low weight, and compatibility with most liquids and lubricants. They are also sometimes biocompatible or biodegradable. In order to increase some of their mechanical properties these materials can be reinforced by adding small particles or fibers. Since Hunter (1961), it is well known that (frictionless) rolling over a viscoelastic material induces an apparent friction coefficient. This apparent friction has been studied by several authors in the case of homogeneous viscoelastic solid, mostly in the steady-state regime.

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Nomenclature

Letters

a^*	contact radius
a_1, a_2, a_3	semi-axes of an ellipsoidal inhomogeneity
B_{ijkl}	influence coefficients that relating the stress σ_{ij} at point (x_1, x_2, x_3) to the constant eigenstrain at the point (x_1^k, x_2^k, x_3^k)
C_{ijkl}^M, C_{ijkl}^I	elastic constants of the matrix and the inhomogeneity
E^I	Young's modulus of the inhomogeneity
h	distance between the two surfaces of the contacting bodies
$H(t)$	the Heaviside step function
I_{ijkl}	the fourth-order identity tensor
$J(t)$	viscoelastic creep function
K^n	coefficients in the normal displacement at the contact surface due to the contact pressure
M_{ij}	influence coefficients relating the stress σ_{ij} at the point (x_1, x_2, x_3) to the normal traction σ^n within a discretized area centered at $(x_1^k, x_2^k, 0)$
n_1, n_2, n_3	grid sizes in the half-space along the Cartesian directions x_1, x_2, x_3, respectively
P_0	maximum Hertzian pressure
p	contact pressure distribution
D	indenter diameter
$R(t)$	viscoelastic relaxation function
S_{ijkl}	components of Eshelby's tensor
u_i^0	displacements corresponding to the infinite applied strain ε_{ij}^0
u_i	disturbed contribution of the displacements
W	applied exterior load
dx_3	depth of the inclusion from the surface of the matrix in EF model
$x^I = (x_1^I, x_2^I, x_3^I)$	Cartesian coordinates of the inclusion center

Greek letters

ε_{ij}^0	infinite applied strain
ε_{ij}	strain due to eigenstrains
ε_{ij}^*	eigenstrain due to the presence of inhomogeneities
σ_{ij}^0	stress corresponding to the infinite applied strain ε_{ij}^0
σ_{ij}	disturbed contribution of the stresses
ϕ, Ψ	harmonic and biharmonic potentials of mass density ε_{ij}^*
δ_{ij}	Kronecker symbol
σ^n	normal pressure due to the summation of both symmetric inclusions
Δx_1, Δx_2	half-size of the discretized surface area
ν^M, ν^I	Poisson's ratio of the matrix M and the inclusion I
γ	the ratio of the inhomogeneity Young's modulus to the matrix
η	the dashpot viscosity
τ	the relaxation time
θ	the tilted angle of the inhomogeneity in the x_1Ox_3 plane

Acronyms and fast Fourier transforms

2D-FFT	two-dimensional fast Fourier transform
3D-FFT	three-dimensional fast Fourier transform
FFT^{-1}	inverse FFT operation
$\widehat{B}_{ijkl}$	frequency response of coefficients B_{ijkl} in the frequency domain
$\widehat{M}_{ij}$	frequency response of coefficients M_{ij} in the frequency domain

The first works on viscoelastic contact focused on the indentation problem between a rigid indenter and a viscoelastic solid. Lee and Radok (1960) obtained the contact pressure distribution for the spherical indentation of a linear viscoelastic material for a monotonic increase of the contact area. Their model has been extended by Hunter (1960) and Graham (1967) to the indentation of viscoelastic materials when the contact radius possesses a single maximum. More recently Greenwood (2010) introduced a model to solve the contact problem between an axisymmetric indenter and a viscoelastic half-space. The analytical solution of the rebound indentation problem for a linear viscoelastic layer has been given by Argatov (2012). One of the main limitations of all the approaches mentioned above is their restriction to ideal linear viscoelastic materials with one relaxation time. Chen et al. (2011) recently introduced a robust semi-analytical approach to solve indentation problems between a rigid indenter and a homogeneous viscoelastic half-space. The semi-analytical approach allows to account for a wide spectra of relaxation times for linear viscoelastic materials, an arbitrary loading profile and can also be used to simulate the contact between rough surfaces. The semi-analytical approach was recently extended to solve the indentation problem between a rigid indenter and a heterogeneous viscoelastic material (Koumi et al., 2014a). The model can account for the presence of isotropic or anisotropic elastic ellipsoidal inhomogeneities of any orientation within the viscoelastic matrix.

Other authors investigated the steady-state response of the frictionless rolling contact problem when one of the bodies in contact is homogeneous and viscoelastic. The two-dimensional contact problem of a rigid cylinder rolling over a viscoelastic half-space was first solved by Hunter (1961). His plane strain model was limited to an ideal viscoelastic material with one relaxation time. Later, Panek and Kalker (1980) extended Hunter's approach to three-dimensional problems by using an approximation based on the elastic line contact theory (Kalker, 1972, 1977; Panek and Kalker, 1977). This assumption represents a very strong approximation. Goriacheva (1973) used another approach to solve the rolling contact problem between a cylindrical rigid indenter and viscoelastic halfspace. The three-dimensional sliding contact problem of a smooth indenter and a viscoelastic halfspace has been solved later by Aleksandrov et al. (2010). The contact pressure distribution and the resulting torque are presented in the case of an ideal viscoelastic material with one relaxation time. Persson (2010) presented a new analytical theory for the rolling contact