Journal of the Mechanics and Physics of Solids 61 (2013) 2047–2066

Contents lists available at SciVerse ScienceDirect

Journal of the Mechanics and Physics of Solids

journal homepage: www.elsevier.com/locate/jmps

Modeling of dielectric elastomers: Design of actuators and energy harvesting devices

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ARTICLE INFO

Article history:
Received 11 September 2012
Received in revised form
19 January 2013
Accepted 12 May 2013
Available online 27 May 2013

Keywords:
Dielectric elastomers
Large deformations
Actuators
Energy harvesting devices
Finite-element method

ABSTRACT

Dielectric elastomers undergo large deformations in response to an electric field and consequently have attracted significant interest as electromechanical transducers. Applications of these materials include actuators capable of converting an applied electric field into mechanical motion and energy harvesting devices that convert mechanical energy into electrical energy. Numerically based design tools are needed to facilitate the development and optimization of these devices. In this paper, we report on our modeling capability for dielectric elastomers. We present the governing equations for the electromechanically coupled behavior of dielectric elastomers in a thermodynamic framework and discuss the attendant finite-element formulation and implementation, using a commercial finite-element code. We then utilize our simulation capability to design and optimize complex dielectric elastomeric actuators and energy harvesting devices in various settings.

1. Introduction

Dielectric elastomers, or soft dielectrics, are rubbery materials that undergo large elastic deformations in response to an applied electric field (cf. e.g. Pelrine et al., 1998; Zhang et al., 1998; Plante and Dubowsky, 2006; Hossain et al., 2012; Tagarielli et al., 2012). These materials were first reported by Pelrine and coworkers (Pelrine et al., 1998, 2000a,b; Kornbluh et al., 2000) and have since then garnered interest as electromechanical transducers for a wide variety of applications (Carpi et al., 2008), such as robotics, biomedical engineering and energy harvesting. Their capacity for large, reversible deformations distinguishes dielectric elastomers from more conventional electromechanical transducers, such as piezoelectrics. Soft dielectrics are also comparatively lighter, more compliant and less expensive, increasing their appeal.

At a microscopic level, dielectric elastomers are electrically insulating, polymeric materials made up of long-chain molecules, possessing charge imbalances that align, or polarize, in the presence of an electric field. Since these materials can achieve both large mechanical deformations and electrically polarize, they are capable of converting energy between mechanical and electrical forms. In particular, actuators that convert an applied electric field into mechanical motion have been used in robotics, including as artificial muscles for biomimetic robots and prosthetics (Bar-Cohen, 2001; Brochu and Pei, 2010; Carpi et al., 2005; Carpi and Smela, 2009). In contrast, soft dielectric devices that convert mechanical energy into electrical energy are referred to as energy harvesting devices (Kornbluh et al., 2011, 2012). In these applications, mechanical energy from motion such as human walking or ocean waves is converted to electrical energy and stored. These technologies

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0022-5096/$ - see front matter
http://dx.doi.org/10.1016/j.jmps.2013.05.003

have been demonstrated in practice; however, the development of rigorous design tools that may be used for optimization in varied applications remains a challenge.

A design capability for dielectric elastomers requires (i) a finite-deformation constitutive theory for the electromechanically coupled response of these materials and (ii) a robust numerical implementation of the resulting field equations. Work on electromechanically coupled constitutive theories goes back many decades (Toupin, 1956, 1963; Maugin, 1980, 1988; Eringen and Maugin, 1990; Maugin et al., 1992; Goulbourne et al., 2005; Dorfmann and Ogden, 2005; Bustamante et al., 2009), and in recent years the mechanics community has come to a relative agreement regarding the formulation of a theory in a thermodynamic framework (McMeeking and Landis, 2005; McMeeking et al., 2007; Suo et al., 2008; Zhao et al., 2007; Zhao and Suo, 2008a). However, there remains a need for numerical tools capable of predicting the large-deformation, three-dimensional, coupled response. Several approaches have been undertaken. First, simplified finite-element computational procedures have been proposed that make geometrical assumptions, reducing the electrical problem to one dimension (Wissler and Mazza, 2005, 2007; Zhao and Suo, 2008b; Zhou et al., 2008). These techniques are useful for basic actuator configurations; however, fully three-dimensional procedures are needed to guide the design of more complex devices. Recently, finite-element implementations have been reported using in-house codes both in quasi-static (Vu and Steinmann, 2007; Vu et al., 2007) and dynamic settings (Park et al., 2012); however, these codes are not available to the community. Given the industrial and scientific community's growing interest in dielectric elastomers, implementation of the theory within a widely available finite-element software is a crucial step toward facilitating interactions between industry and researchers and guiding the design of complex three-dimensional devices. Unfortunately, this task is not straightforward within commercial finite-element packages, since additional nodal degrees of freedom are required. Few efforts in this direction have been reported, namely using FEAP (Gao et al., 2011) and Comsol (Rudykh and deBotton, 2012). However, since FEAP is a general-purpose finite-element program, designed for research and educational use, it is not available to the industrial community. Moreover, although Comsol is amenable to the implementation of the coupled electromechanical theory, its difficulty in dealing with large deformations is well-known, and as such, it is not well-suited for problems involving dielectric elastomers.

To overcome these issues, the purpose of this paper is threefold: (i) to present a concise thermodynamic development of the three-dimensional, fully coupled theory governing the behavior of dielectric elastomers, (ii) to implement the theory in the commercial finite-element code Abaqus/Standard (2010), taking full advantage of the capability to actively interact with the software through user-defined subroutines, and (iii) to utilize the code to provide new insights, through simulation, into the design and optimization of complex actuators and energy harvesting devices in various settings. Abaqus is an attractive platform because it is a well-known code, widely available, stable, portable, and particularly suitable for analyses involving large deformations. The novelty of the present work is in its completeness, encompassing theoretical formulation, numerical implementation, and application to design and optimization. We expect our simulations of actuators and energy harvesting devices to aid in improving upon the designs of these structures. Further, our Abaqus user-defined subroutines and input files may be found online as supplementary material to be used and expanded upon by the community in further research on dielectric elastomers.

The paper is organized as follows. In Section 2, we lay out the continuum framework used to address the electromechanical behavior, and in Section 3, we specify specific constitutive equations for a representative dielectric elastomer. In Section 4, we review the resulting boundary-value problem and its finite-element implementation. Finally, in Section 5, we verify our finite-element implementation and demonstrate its application by simulating the operation of several complex actuators and energy harvesting devices made from dielectric elastomers.

2. Continuum framework

In this section, we summarize the equations governing the nonlinear, electrostatic deformation of soft dielectrics, following the formulations previously introduced by McMeeking and Suo and their coworkers (McMeeking and Landis, 2005; McMeeking et al., 2007; Suo et al., 2008; Zhao et al., 2007; Zhao and Suo, 2008a).

Kinematics. We consider a homogeneous body $\mathcal{B}_R$ identified with the region of space it occupies in a fixed reference configuration, and denote by $\mathbf{x}_R$ an arbitrary material point of $\mathcal{B}_R$. A motion of $\mathcal{B}_R$ is then a smooth one-to-one mapping $\mathbf{x}=\chi(\mathbf{x}_R,t)$ with deformation gradient given by[1]

$$\mathbf{F}=\nabla\chi, \tag{2.1}$$

such that $J=\det\mathbf{F}>0$. The right and left and polar decompositions of $\mathbf{F}$ are given by $\mathbf{F}=\mathbf{RU}=\mathbf{VR}$, where $\mathbf{R}$ is a rotation (proper orthogonal tensor), while $\mathbf{U}$ and $\mathbf{V}$ are symmetric, positive-definite stretch tensors. Also, the right and left Cauchy-Green tensors are given by $\mathbf{C}=\mathbf{U}^2=\mathbf{F}^\top\mathbf{F}$ and $\mathbf{B}=\mathbf{V}^2=\mathbf{F}\mathbf{F}^\top$, respectively.

Electric potential, electric field and Faraday's law. Central to the discussion of dielectric elastomeric materials is the electric potential $\varphi(\mathbf{x}_R,t)$. We define the referential electric field as

$$\mathbf{E}_R\overset{\text{def}}{=}-\nabla\varphi, \tag{2.2}$$

[1] The symbols ∇, Div and Curl denote the gradient, divergence and curl respectively with respect to the material point $\mathbf{x}_R$ in the reference configuration; grad, div and curl denote these operators with respect to the point $\mathbf{x}=\chi(\mathbf{x}_R,t)$ in the deformed configuration.