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Capillary buckling of a thin film adhering to a sphere

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ABSTRACT

We present a combined theoretical and experimental study of the buckling of a thin film wrapped around a sphere under the action of capillary forces. A rigid sphere is coated with a wetting liquid, and then wrapped by a thin film having an approximately cylindrical shape. The equilibrium of the film is governed by the competing effects of elasticity and capillarity: elasticity tends to keep the film developable while capillarity tends to curve it in both directions so as to maximize the area of contact with the sphere. In the experiments, the region of contact between the film and the sphere has cylindrical symmetry when the sphere radius is small, but destabilizes to a non-symmetric, wrinkled configuration when the radius is larger than a critical value. We combine the Donnell equations for near-cylindrical shells to include a unilateral constraint with the impenetrable sphere, and the capillary forces acting along a moving edge. A non-linear solution describing the axisymmetric configuration of the film is derived. A linear stability analysis is then presented, which successfully captures the wrinkling instability, the symmetry of the unstable mode, the instability threshold and the critical wavelength. The motion of the free boundary at the edge of the region of contact, which has an effect on the instability, is treated without any approximation.

1. Introduction

The buckling of thin plates has been studied for a long time, both theoretically (Timoshenko and Woinowski-Krieger, 1959; Timoshenko and Gere, 2009) and experimentally (Singer et al., 1997). Initially, the main motivation was to avoid loads associated with catastrophic failure modes. Recent research efforts on thin plates and shells have been driven by technological applications involving thinner and thinner plates (Geim and Novoselov, 2007), by the idea that controlled buckling can provide useful functionality (Shim et al., 2012), and by strongly non-linear phenomena appearing far above the bifurcation threshold (Davidovitch et al., 2011). The wrinkling of a semi-infinite elastic medium under finite compression, known as Biot's problem (Hohlfeld and Mahadevan, 2011; Cao and Hutchinson, 2011), as well as the buckling of a thin stiff film coated to a compliant substrate (Chen and Hutchinson, 2004; Audoly and Boudaoud, 2008; Cai et al., 2011) are just two examples of classical problems in mechanical engineering whose non-linear aspects have been well understood only recently. We refer the reader to Li et al. (2012) for a comprehensive review.

When the adhesion between a thin film under residual compression and a thick substrate is relatively weak, buckling can take place along with delamination, resulting in the formation of blisters (Gioia and Ortiz, 1997; Vella et al., 2009). Buckling-driven delamination can lead to various patterns which are affected both by the mode-mixity (Hutchinson and

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Suo, 1992), i.e. the dependence of the interfacial energy on the loading mode, and by the irreversibility of the interfacial fracture (Audoly, 2000). In recent experiments, a simpler variant of the classical delamination problem has been proposed, whereby the adhesion between the film and the substrate is provided by capillary forces arising from a thin liquid bridge (Bico et al., 2004; Hure et al., 2011): capillary forces are reversible and act like a self-healing interface crack. These experiments can be done at the centimeter scale. Broadly speaking, there is a recent surge of interest for systems driven by the interplay of elastic and capillary forces, which are dominant at small scales. Applications range from metrology (Huang et al., 2007) to the fabrication of 3d photovoltaic receptor (Guo et al., 2009) featuring increased efficiency, to cite just a few. The swelling of plates is another example of buckling that can be used at small scales to design 3d shapes out of patterned 2d surfaces (Klein et al., 2007; Kim et al., 2012), with applications to soft actuators and robots (Lee et al., 2010).

In the present paper, we study some buckling patterns produced by elastocapillary delamination experiments. Specifically, we consider the case of the capillary adhesion between a thin elastic film and a doubly curved substrate. This geometry has been considered by one of us in a recent experimental paper (Hure et al., 2011). It built up on previous works addressing the related case of a spherical shell adhering onto a planar substrate (Tamura et al., 2004; Komura et al., 2005; Springman and Bassani, 2008). In our experiments, a rigid spherical cap is first coated by a wetting liquid and a thin polypropylene film is then applied onto it. As it wraps the sphere under the action of capillary forces, the film is forced to stretch by Gauss' *theorema egregium* (Struik, 1988). Stretching allows it to make up for the mismatch of Gaussian curvature, which is zero in the planar film and non-zero along the spherical substrate. This leads to a variety of adhesion morphologies, as shown in Fig. 1, where the contact region varies from a simple band to complex branched patterns.

In Hure et al. (2011), one of us studied the competing effects of elasticity and capillarity using order of magnitude arguments, and proposed an estimate for the size of the region of adhesion which successfully compares to the experiments.

Here, we study these patterns quantitatively. In particular we address the transition shown in the figure, whereby a band-like region of contact with straight edges (Fig. 1a) bifurcates into a sinuous pattern with undulatory edges (Fig. 1b) when the adhesion becomes stronger or the film becomes thinner. This is interpreted as a buckling bifurcation caused by compressive stress along the straight edges. We carry out a stability analysis based on the classical Donnell equations for nearly cylindrical shells, modified to account for the effect of adhesion. The motion of the free boundary at the edge of the region of contact is considered without any approximation.

The paper is organized as follows. In Section 2, we derive the equations for a nearly cylindrical elastic shell adhering to a sphere, with an emphasis on the equilibrium conditions along the edge of the moving contact region. In Section 3, we derive a non-linear solution to these equations relevant to the unbuckled configuration with cylindrical symmetry. These results are compared to experimental data. In Section 4, we then study the linear stability of the cylindrical solution. The predictions regarding the symmetry of the buckling modes, their wavelength and the critical loads are compared to experimental data in Section 5.

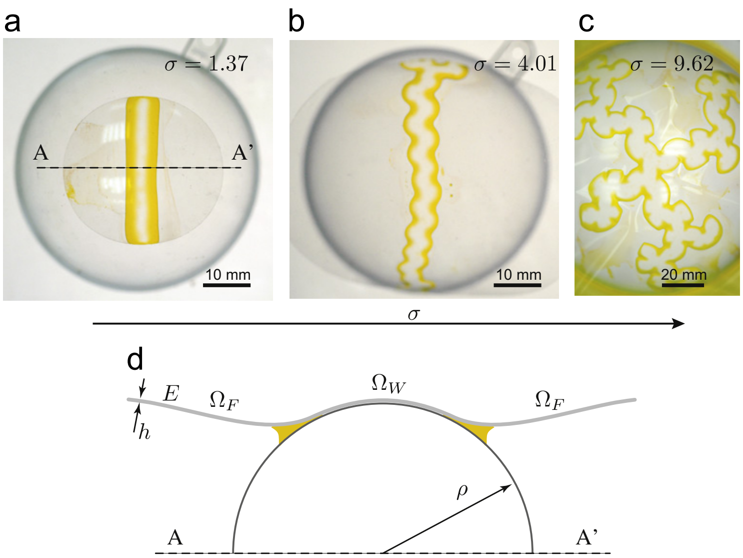

Fig. 1. Thin elastic films of thickness h, Young's modulus E and Poisson's ratio ν are placed onto rigid spherical caps of radius ρ coated with a wetting liquid, with surface tension $\gamma = 22.4\ \mathrm{mN\ m^{-1}}$. The liquid has been dyed in yellow to help visualization: the yellow regions are the liquid meniscus. Note that the interior of the region of contact appear uncolored as there is almost no fluid there. Top view of three different experiments: (a) $E = 2.8$ GPa, $h = 30\ \mu$m, $\rho = 25$ mm (b) $E = 2.6$ GPa, $h = 15\ \mu$m, $\rho = 25$ mm (c) $E = 2.6$ GPa, $h = 15\ \mu$m, $\rho = 60$ mm. The parameter σ, defined in Eq. (19), measures the strength of adhesion relative to the stiffness of the film. (d) Sketch of a cut through a vertical plane AA′. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)