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Self-equilibrium and stability of regular truncated tetrahedral tensegrity structures

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ABSTRACT

This paper presents analytical conditions of self-equilibrium and super-stability for the regular truncated tetrahedral tensegrity structures, nodes of which have one-to-one correspondence to the tetrahedral group. These conditions are presented in terms of force densities, by investigating the block-diagonalized force density matrix. The block-diagonalized force density matrix, with independent sub-matrices lying on its leading diagonal, is derived by making use of the tetrahedral symmetry via group representation theory. The condition for self-equilibrium is found by enforcing the force density matrix to have the necessary number of nullities, which is four for three-dimensional structures. The condition for super-stability is further presented by guaranteeing positive semi-definiteness of the force density matrix.

1. Introduction

Self-equilibrium analysis and stability properties are the two key problems to designing a tensegrity structure, both of which can be dealt with by investigating the eigenvalues of force density matrix (Zhang and Ohsaki, 2007). In this study, we analytically decompose the force density matrix into independent sub-matrices (blocks), and then derive the conditions for their self-equilibrium as well as super-stability, by making use of the high tetrahedral symmetry of the regular truncated tetrahedral structures.

An example of the structure interested in this study is shown in Fig. 1. This kind of tensegrity structures was invented by Fuller (1962). The structure consists of 12 nodes and 24 members, including 6 struts in compression and 18 cables in tension. The cables lie along the edges of a truncated tetrahedron, which is made by cutting off the vertices of a tetrahedron; and the struts are the diagonals connecting the vertices of the truncated tetrahedron.

There are many other tensegrity structures with tetrahedral symmetry, for example the structures achieved by truncating the vertices of a truncated tetrahedron using the polyhedral truncation scheme by Li et al. (2010). In this study, we will concentrate only on the structures with nodes having one-to-one correspondence to the symmetry operations of a tetrahedral group.

Let p, q, m, and n, respectively, denote number of independent prestress modes, number of infinitesimal mechanisms, number of members, and number of nodes. For the study of stability of a pin-jointed (truss-like) structure, Calladine

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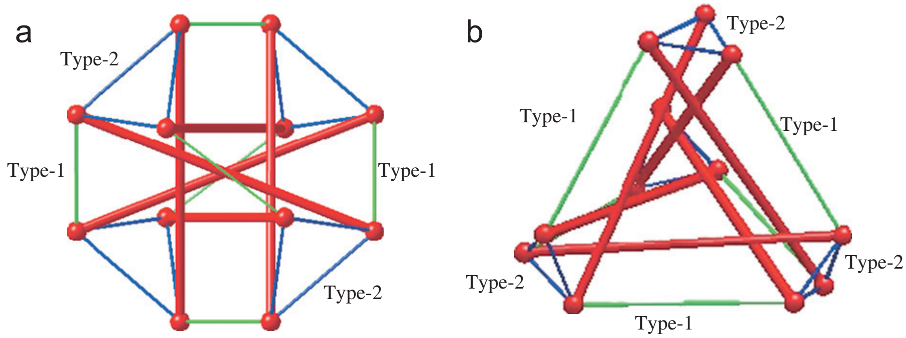

Fig. 1. A tensegrity structure with tetrahedral symmetry. The thinner members are cables in tension, and thicker members are struts in compression. The cables lie on the edges, and the struts connect the vertices of a regular truncated tetrahedron: (a) top view, (b) side view.

(1978) presented a modified version of Maxwell's (1864) rule as follows:

$$p - q = m - 3n(+6), \tag{1}$$

where the number 6 on the right-hand side disappears if rigid-body motions are constrained. The q's being positive is the necessary condition for stability of a conventional pin-jointed structure, in the sense of having positive-definite tangent stiffness.

Tensegrity structures are free-standing, without any support to constrain the rigid-body motions. Thus, for the tensegrity structure in Fig. 1, Eq. (1) gives

$$p - q = 24 - 3 \times 12 + 6 = -6, \tag{2}$$

where the minus sign indicates that the structure cannot be *stable*, if no prestress is introduced. However, as will be proved later in this study, the tetrahedral symmetry version of this structure can be stable, and even *super-stable*, if it contains proper prestresses. When a structure is said to be super-stable, it is always stable irrespective of material properties as well as level of prestress (Connelly and Back, 1998; Zhang and Ohsaki, 2007).

The structure has only one mode of prestress; i.e., $p=1$, thus, there exists in total seven infinitesimal mechanisms; i.e., $q=7$. It is this single prestress mode that can stiffen the seven infinitesimal mechanisms to guarantee stability of the structure.

To find out the prestress mode, Raj and Guest (2006) considered the structures that are of tetrahedral symmetry to simplify their investigations. Making use of the high symmetry, they derived the symmetry-adapted form of the force density matrix by a semi-analytical approach, where the matrix is block-diagonalized to have its independent sub-matrices lying on the diagonal. The curves of solutions, in terms of force densities, are derived and plotted by enforcing the 3-by-3 sub-matrix to be singular. Furthermore, a short discussion on condition for super-stability of this kind of structures is given based on the plotted solution curves.

For the same problem, Tsuura et al. (2010) conducted self-equilibrium analysis and derived the same solutions. The analytical condition for self-equilibrium is derived by considering the equilibrium of only one node, due to the high symmetry. The condition for super-stability has also been presented, though also in a *numerical and illustrative* way.

In the previous study (Zhang et al., 2009b), the authors presented an *analytical* formulation for block-diagonalizing force density matrices of the tensegrity structures that are of dihedral symmetry, via group representation theory. The self-equilibrium and super-stability investigations significantly simplified, and the derivation of analytical conditions become possible. The approach has been applied to the prismatic tensegrity structures (Zhang et al., 2009a) as well as star-shaped tensegrity structures (Zhang et al., 2010), both of which are of dihedral symmetry.

In this paper, we are to extend the approach to the structures with tetrahedral symmetry, to present analytical conditions for their self-equilibrium and super-stability.

Following this introductory section, the paper is organized as follows: Section 2 describes configuration and symmetry properties of the structures with tetrahedral symmetry; Section 3 formulates the analytical symmetry-adapted form of the force density matrix; Section 4 finds the self-equilibrium prestress modes, in terms of force densities; Section 5 presents the super-stability conditions, also in terms of force densities; Section 6 demonstrates self-equilibrium configurations of several example structures; and Section 7 briefly concludes the study.

2. Tetrahedral symmetry

Symmetry of a structure can be systematically dealt with by using group representation theory. In particular, the theory on irreducible representation matrices is important for us to derive the symmetry-adapted form of the force density matrix, which is used for the implementation of self-equilibrium analysis and stability investigation in the following sections. Thus, this section gives a brief introduction to tetrahedral group and its irreducible representation matrices. More