Studies in History and Philosophy of Modern Physics 50 (2015) 54–69

Contents lists available at ScienceDirect

Studies in History and Philosophy of Modern Physics

journal homepage: www.elsevier.com/locate/shpsb

Local reduction in physics

Joshua Rosaler

University of Minnesota, Center for Philosophy of Science, Heller Hall, 271 19th Ave S 55455, Minneapolis, MN, United States

ARTICLE INFO

Article history:
Received 6 May 2014
Received in revised form
29 January 2015
Accepted 23 February 2015
Available online 24 April 2015

Keywords:
Reduction
Local Reduction
Models
Dynamical systems
Nagel
Limits

ABSTRACT

A conventional wisdom about the progress of physics holds that successive theories wholly encompass the domains of their predecessors through a process that is often called "reduction." While certain influential accounts of inter-theory reduction in physics take reduction to require a single "global" derivation of one theory's laws from those of another, I show that global reductions are not available in all cases where the conventional wisdom requires reduction to hold. However, I argue that a weaker "local" form of reduction, which defines reduction between theories in terms of a more fundamental notion of reduction between *models* of a single fixed system, is available in such cases and moreover suffices to uphold the conventional wisdom. To illustrate the sort of fixed-system, inter-model reduction that grounds inter-theoretic reduction on this picture, I specialize to a particular class of cases in which both models are dynamical systems. I show that reduction in these cases is underwritten by a mathematical relationship that follows a certain liberalized construal of Nagel/Schaffner reduction, and support this claim with several examples. Moreover, I show that this broadly Nagelian analysis of inter-model reduction encompasses several cases that are sometimes cited as instances of the "physicist's" limit-based notion of reduction.

When citing this paper, please use the full journal title *Studies in History and Philosophy of Modern Physics*

1. Introduction

According to the most commonly told story about the progress of physics, successive theories in physics come ever closer to revealing the true, fundamental nature of reality. This convergence rests on the supposition that later theories bear a special relationship to their predecessors often called "reduction," which minimally requires one theory to encompass the domain of application of another. More specifically, the conventional wisdom tells us that Newtonian mechanics "reduces to" special relativity,[1] special relativity to general relativity, classical mechanics to quantum mechanics, quantum mechanics to relativistic quantum mechanics, relativistic quantum mechanics to quantum field theory, thermodynamics to statistical mechanics, and more. In order to assess the truth of the conventional wisdom, however, it is necessary to gain a more precise sense of what is needed in a given case to show that one theory reduces to another.

In his widely cited 1973 paper, Nickles distinguished two types of approach to reduction in physics: first, the approach commonly employed by philosophers, which originates in Ernest Nagel's well-known account of reduction, and second, the approach commonly employed by physicists that requires one theory to be a "limit" or "limiting case" of another (Nickles, 1973). Since Nickles' paper, these two accounts have tended to dominate philosophical discussion concerning issues of the general methodology of reduction in physics. As commonly presented, both strongly suggest—and in some cases, state explicitly—that reduction between theories in physics should rest on a single "global" derivation of a high-level theory's laws from those of a low-level theory. Here, I argue by means of a particular example that global reduction is not always available in cases where the conventional wisdom requires reduction to hold. However, I argue that it is possible to a define a weaker "local" notion of reduction in physics that suffices to uphold the conventional wisdom in that it suffices to ensure the subsumption of one theory's domain by another. This

[1] As Nickles noted several decades ago, two opposing conventions have arisen in the literature on inter-theoretic reduction, one (employed most commonly in the philosophical literature) that takes a less encompassing theory to "reduce to" a more encompassing one, and the other (employed most commonly in the physics literature) that takes the more encompassing theory to "reduce to" the less encompassing one (Nickles, 1973). Here, I will adopt the first of these conventions. It should also be noted that the two concepts of reduction that Nickles discusses in his paper differ on more substantive points than this choice of convention, which I discuss further below.

http://dx.doi.org/10.1016/j.shpsb.2015.02.004
1355-2198/

notion of reduction is "local" in the sense that it permits the reducing theory to account for the reduced theory's success through numerous context-specific derivations that are relativized to different systems in the high-level theory's domain. These derivations concern the specific *models* that the theories use to describe a single fixed system, rather than the theories as a whole.

This paper has two main goals, which are mutually supporting. The first is to motivate and develop a local account of inter-theoretic reduction in physics. Inter-theoretic reduction in physics, understood minimally as the requirement that one theory subsume the domain of another, does not require anything as strong as global reduction directly between theories; local reduction suffices, and moreover avoids difficulties that afflict global approaches. I further argue that local reduction between theories should be understood in terms of the more basic notion of reduction between models of a single fixed system. The second goal of the paper is to give an account of fixed-system, inter-model reduction—on which this local account of inter-theoretic reduction rests—in a specialized class of cases where both models of the system in question are dynamical systems, and in particular to show that such cases can be analyzed in terms of a certain local, model-based adaptation of the Nagel/Schaffner approach to reduction. I further show that this Nagelian analysis of inter-model reduction encompasses many cases that have been cited as instances of physicists' limit-based notion of reduction, as well as providing a more precise characterization of these cases than do existing formulations of the limit-based approach.

The present analysis of reduction is given in two parts, corresponding respectively to the two goals just described. Part I, which consists of Sections 2 and 3, is largely non-technical and concerns issues of general methodology. As suggested, its purpose is to motivate and present a certain local, model-based approach to inter-theoretic reduction and to explain how this strategy avoids certain difficulties that afflict more global approaches. In Section 2, I briefly review two approaches to reduction—global Nagelian and global limit-based—that are often taken as the focus of philosophical discussions on this topic, and highlight some of their limitations. In Section 3, I sketch a local approach to inter-theoretic reduction in physics that relies on the more basic notion of fixed-system reduction between models and respond to one major objection that such an approach is likely to elicit.

Part II, which consists of Sections 4–6, provides a detailed technical analysis of fixed-system, inter-model reduction in a particular set of cases where both models of the system in question are dynamical systems, as well as briefly discussing various possible expansions of this analysis to a more comprehensive account of fixed-system, inter-model reduction in physics. Section 4 describes a general mathematical relationship between dynamical systems models that serves to underwrite many real instances of fixed-system, inter-model reduction in physics. In a certain strong sense, this mathematical relationship constitutes an application of the criteria for Nagel/Schaffner reduction to the context of fixed-system, inter-model reduction between dynamical systems models. Section 5 shows how this general relationship serves to characterize reduction across a wide range of particular cases, and to subsume a number of cases that are commonly cited as examples of the "physicist's" limit-based notion of reduction. Section 6 briefly discusses possibilities for extending and generalizing this strategy for inter-model reduction beyond the set of cases discussed here: first, to an analysis of the relationship between symmetries of the two models involved in a reduction, and second, to an analysis of cases where one or both of the models involved in the reduction is not a dynamical system but some other kind of model (e.g., stochastic, non-dynamical, etc.).

The distinct portions of the analysis given in Parts I and II complement each other in a number of important ways. Part I serves to frame the analysis of reduction between dynamical systems given in Part II within a more general picture of inter-theoretic reduction and in particular to situate this analysis relative to the two accounts of inter-theoretic reduction in physics first distinguished by Nickles. By the same token, Part II provides a concrete illustration of the sort of fixed-system, inter-model reduction that is taken as the basis for the local approach to inter-theoretic reduction described in Part I.

1.1. A few points of terminology

Before proceeding, it is worth taking a moment to clarify several points of terminology.

Because debates about reduction are often frought with ambiguity as to what, precisely, is meant by reduction, I should clarify my use of the term here. I do not attach my usage to any specific account of reduction—for example, Nagelian, limit-based, New Wave and functionalist approaches. Rather, I use it to designate a certain general concept that, I take it, all, or most, of the many specific accounts aim to make more precise. "Reduction," then, is taken to designate the general requirement that two descriptions of the world "dovetail" in such a manner that one description entirely encompasses the range of successful applications of the other. That is, reduction on this usage requires *subsumption* of one description's domain of applicability by the other, while the specific sense in which the two descriptions "dovetail" in order to achieve this is deliberately left vague, so as not to bias its usage toward any particular account.

As Nickles has noted, the usage of the term "reduction" most common among philosophers takes the less accurate and encompassing description in a reduction to "reduce to" the more accurate and encompassing description, whereas the usage most common among physicists takes the more accurate, encompassing description to "reduce to" the less accurate and encompassing description. In what follows, I will always adopt the philosopher's convention, even when discussing the physicist's limit-based notion of reduction, so that if theory T_2 is a "limiting case" of T_1, we will say that T_2 "reduces to" T_1.

I will also reserve the term "high-level" to refer to the description that is purportedly reduced and "low-level" to refer to the description that purportedly does the reducing. This usage generalizes another use of the "high-level/low-level" distinction, which presupposes that the high-level description is in some sense a coarse-graining of the low-level description, or that the high-level description is in some sense "macro" and the low-level description in some sense "micro." Here, no such assumption is made. For example, where the relation between Kepler's and Newton's theories of planetary motion is concerned, Kepler's theory would count on our usage as the "high-level" and Newton's as the "low-level" theory even though Kepler's theory is not in any normal sense a coarse-graining of Newton's. While some authors have emphasized the distinction between "inter-level" reductions (e.g., thermodynamics to statistical mechanics) and "intra-level" reduction (e.g., Newtonian mechanics to special relativity, or Kepler's to Newton's theory of planetary motion), the picture of reduction presented here does not rely on this distinction and treats both kinds of reduction on a par.[2]

Henceforth, when I speak of "Nagelian" reduction, the reader should take this to refer specifically to the Nagel/Schaffner account of reduction, which allows for approximative derivations rather than requiring exact derivations. While Nagel/Schaffner reduction is widely framed within a syntactic view of theories—as opposed

[2] As with the term "reduce," it is worth noting that one occasionally finds the high-/low- distinction inverted, so that the "high-level" description is the more encompassing and the "low-level" description the less encompassing of the two.