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Postbuckling analysis and its application to stretchable electronics

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ABSTRACT

A versatile strategy for fabricating stretchable electronics involves controlled buckling of bridge structures in circuits that are configured into open, mesh layouts (i.e. islands connected by bridges) and bonded to elastomeric substrates. Quantitative analytical mechanics treatments of the responses of these bridges can be challenging, due to the range and diversity of possible motions. Koiter (1945) pointed out that the postbuckling analysis needs to account for all terms up to the 4th power of displacements in the potential energy. Existing postbuckling analyses, however, are accurate only to the 2nd power of displacements in the potential energy since they assume a linear displacement–curvature relation. Here, a systematic method is established for accurate postbuckling analysis of beams. This framework enables straightforward study of the complex buckling modes under arbitrary loading, such as lateral buckling of the island-bridge, mesh structure subject to shear (or twist) or diagonal stretching observed in experiments. Simple, analytical expressions are obtained for the critical load at the onset of buckling, and for the maximum bending, torsion (shear) and principal strains in the structure during postbuckling.

1. Introduction

Recent work suggests that it is possible to configure high performance electronic circuits, conventionally found in rigid, planar formats, into layouts that match the soft, curvilinear mechanics of biological tissues (Rogers et al., 2010). The resulting capabilities open up many application opportunities that cannot be addressed using established technologies. Examples include ultralight weight, rugged circuits (Kim et al., 2008a), flexible inorganic solar cells (Yoon et al., 2008) and LEDs (Park et al., 2009), soft, bio-integrated devices (Kim et al., 2010a, 2010b, 2011a, 2011b; Viventi et al., 2010), flexible displays (Crawford, 2005), eye-like digital cameras (Jin et al., 2004; Ko et al., 2008; Jung et al., 2011), structural health monitoring devices (Nathan et al., 2000), and electronic sensors for robotics (Someya et al., 2004; Mannsfeld et al., 2010; Takei et al., 2010) and even 'epidermal' electronics capable of mechanically invisible integration onto human skin (Kim et al., 2011b). Work in this area emphasizes mechanics and geometry, in systems that integrate hard materials for the active components of the devices with elastomers for the substrates and packaging components. (Rogers et al., 2010).

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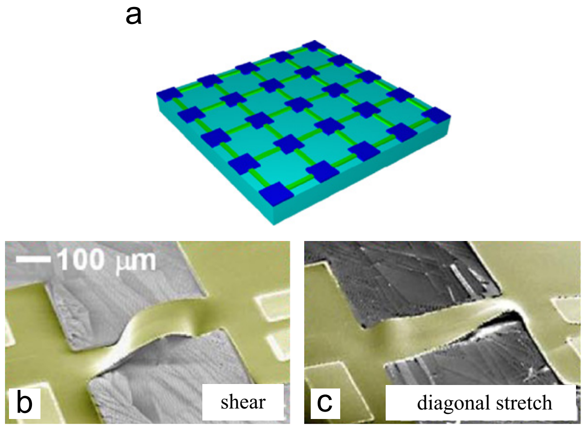

Fig. 1. (a) A schematic diagram of island-bridge, mesh structure in stretchable electronics; (b) SEM image (colorized for ease of viewing) of the island-bridge, mesh structure under shear; and (c) SEM image of the island-bridge, mesh structure under diagonal stretch.

A particularly successful strategy to stretchable electronics uses postbuckling of stiff, inorganic films on compliant, polymeric substrates (Bowden et al., 1998). Kim et al. (2008b) further improved by structuring the film into a mesh and bonding it to the substrate only at the nodes, as shown in Fig. 1a. Once buckled, the arc-shaped interconnects between the nodes can move freely out of the mesh plane to accommodate large applied strain (Song et al., 2009). The interconnects undergo complex buckling modes, such as lateral buckling, when subject to shear (Fig. 1b) or diagonal stretching (45° from the interconnects) (Fig. 1c). Different from Euler buckling, these complex buckling modes involve large torsion and out-of-plane bending. It is important to ensure the maximum strain in interconnects are below their fracture limit.

Koiter (1945, 1963, 2009) established a robust method for postbuckling analysis via energy minimization. The potential energy was expanded up to 4th power of displacement (from the non-buckled state), and the 3rd and 4th power terms actually governed the postbuckling behavior (Koiter, 1945). The existing analyses for postbuckling of plates (or shells), however, only account for the 1st power of the displacement in the curvature (von Karman and Tsien, 1941; Budiansky, 1973), which translates to the 2nd power of the displacement in the potential energy. This can be illustrated via a beam, whose curvature κ is approximately the 2nd order derivative of the deflection w, i.e., $\kappa = w''$. The bending energy is $(EI/2)\int w''^2 dx$, where EI is the bending stiffness, and the integration is along the central axis x of the beam. The accurate expression of the curvature is $\kappa = w''/(1+w'^2)^{3/2} \approx w''[1-(3/2)w'^2]$, which gives bending energy $(EI/2)\int \kappa^2 dx \approx (EI/2)\int w''^2(1-3w'^2)dx$. It is different from the approximate expression above by $(3EI/2)\int w''^2 w'^2 dx$, which is the 4th power of displacement, and may be important in the postbuckling analysis (Koiter, 1945).

The objective of this work is to establish a systematic method for postbuckling analysis of beams that may involve rather complex buckling modes such as lateral buckling of the island-bridge, mesh structure in stretchable electronics. It avoids the complex geometric analysis for lateral buckling (Timoshenko and Gere, 1961) and establishes a straightforward method to study any buckling mode. It accounts for all terms up to the 4th power of displacements in the potential energy, as suggested by Koiter (1945). Examples are given to show the importance of the 4th power of displacement, including the complex buckling patterns of the island-bridge, mesh structure for stretchable electronics (Fig. 1). Analytical expressions are obtained for the amplitude of, and the maximum strain in, buckled interconnects, which are important to the design of stretchable electronics.

2. Postbuckling analysis

2.1. Membrane strain and curvatures

Let Z denote the central axis of the beam before deformation. The unit vectors in the Cartesian coordinates (X,Y,Z) before deformation are $\boldsymbol{E}_i$ $(i=1,2,3)$. A point $\boldsymbol{X}=(0,0,Z)$ on the central axis moves to $\boldsymbol{X}+\boldsymbol{U}=(U_1,U_2,U_3+Z)$ after deformation, where $U_i(Z)$ $(i=1,2,3)$ are the displacements. The stretch along the axis is

$$\lambda = \sqrt{U_1'^2+U_2'^2+(1+U_3')^2} \approx 1+U_3'+\frac{1}{2}(U_1'^2+U_2'^2)-\frac{1}{2}U_3'(U_1'^2+U_2'^2), \tag{2.1}$$

where $()'=d()/dZ$, and terms higher than the 3rd power of displacement are neglected because their contribution to the potential energy are beyond the 4th power. The length dZ becomes λdZ after deformation.

The unit vector along the deformed central axis is $\boldsymbol{e}_3 = d(\boldsymbol{X}+\boldsymbol{U})/(\lambda dZ)$. The other two unit vectors, $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$, involve twist angle ϕ of the cross section around the central axis. Their derivatives are related to the curvature vector $\boldsymbol{\kappa}$ of the central