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Material microstructure optimization for linear elastodynamic energy wave management

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ABSTRACT

We describe a systematic approach to design material microstructures to achieve desired energy propagation in a two-phase composite plate. To generate a well-posed topology optimization problem we use the relaxation approach which requires homogenization theory to relate the macroscopic material properties to the microstructure, here a sequentially ranked laminate. We introduce an algorithm whereby the laminate layer volume fractions and orientations are optimized at each material point. To resolve numerical instabilities associated with the dynamic simulation and constrained optimization problem, we filter the laminate parameters. This also has the effect of generating smoothly varying microstructures which are easier to manufacture. To demonstrate our algorithm we design microstructure layouts for tailored energy propagation, i.e. energy focus, energy redirection, energy dispersion and energy spread.

1. Introduction

The ability to manage energy propagation has received significant interest in the literature as it has applications in many fields such as impact/blast mitigation, crash worthiness, sound control, earthquake mitigation, etc. It has indeed been shown that acoustic waves can be redirected by utilizing material heterogeneity and anisotropy (Norris and Wickham, 2001; Amirkhizia et al., 2010). In this paper, we use topology optimization to systematically design material microstructures to achieve desired energy propagation. The material microstructure parameters are related to macroscopic material properties, i.e. elasticity tensor and mass density, via homogenization theory. Transient finite element analysis is employed to simulate the energy propagation in macroscopic structures and compute quantities of interests. Sensitivities of the interested quantities with respect to microstructure parameters are computed analytically via an adjoint method and used to iteratively update the design parameters by a gradient-based optimization algorithm. To demonstrate our algorithm, we optimize the material microstructure field of two-phase composite plates, cf. Fig. 1.

In topology optimization two or more material phases are optimally distributed to maximize structural performance, cf. Bendsøe and Sigmund (2003). Since we are designing anisotropic heterogeneous microstructures in an elastodynamics paradigm, topology optimization with relaxation via homogenization (Murat and Tartar, 1985; Kohn and Strang, 1986; Lurie and Cherkaev, 1986; Bendsøe and Kikuchi, 1988; Allaire, 2002; Cherkaev, 2000) is the obvious choice over topology optimization with restriction, e.g. via a solid isotropic material with penalization (SIMP) model (Bendsøe, 1989; Zhou and Rozvany, 1991). As such, the anisotropic heterogeneous microstructure is optimized at each material point to obtain the

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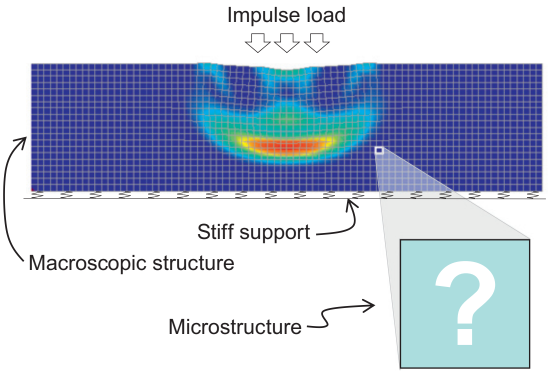

Fig. 1. Optimization problem overview. The contours indicate the total energy at a particular time instant.

most effective use of the constituent materials. This requires knowledge of the $\mathcal{G}$-closure, i.e. the set of achievable homogenized properties for the given phase volume fractions. Unfortunately, this $\mathcal{G}$-closure set of two isotropic phases is unknown in linear elasticity (refer to Bendsøe and Sigmund, 2003, p. 275 and bibliographical notes [4,5,25,34] for further discussion), although bounds on it are known, e.g. the Hashin and Shtrikman (1963) bounds. So instead, one assumes a specific microstructure and optimizes the parameters describing that microstructure, e.g. layer volume fractions and orientations in sequentially ranked laminates (Allaire, 2002; Allaire et al., 2004; Olhoff et al., 1998; Jacobsen et al., 1998; Díaz and Lipton, 1997), or the dimensions and orientation of a rectangular hole in a square cell (Bendsøe and Kikuchi, 1988; Rodrigues and Fernandes, 1995). Alternatively, one can employ a computationally intensive hierarchical approach using inverse homogenization to evaluate the optimal microstructure for the given volume fraction at each material point (Rodrigues et al., 1998, 2002). We use the former, i.e. specific microstructure approach wherein the laminate parameters are assigned and explicit formula (Allaire, 2002; Murat and Tartar, 1997) are used to obtain the homogenized material properties. Ultimately we optimize these laminate parameter fields to obtain the desired macroscopic response.

Dynamic response topology optimization based on frequency domain analysis, i.e. free vibration and forced response using Floquet–Bloch wave theory, has been widely researched to design both micro- and macrostructures. Refer to Bendsøe and Sigmund (2003) bibliographical note [14] for a comprehensive list of references. For example, inverse homogenization has been used to design phononic (elastic wave) (Sigmund and Jensen, 2003; Larsen et al., 2009) and photonic (Cox and Dobson, 2000; Nomura et al., 2009) unit cell microstructures. In other studies multiple materials are optimally distributed in a macroscopic domain to control wave propagation, e.g. Sigmund and Jensen (2003) for elastic, and Jensen and Sigmund (2005) and Frei et al. (2005) for electromagnetic applications. On the other hand, topology optimization for the more computationally intense dynamic structural response based on time domain analysis is less common, e.g. Min et al. (1999), Turteltaub (2005), and Dahl et al. (2008). Related is the optimization of damping distribution using Young measure relaxation in Munch et al. (2006). In this paper, the material microstructural field is optimized to generate the desired time-domain macroscopic energy propagation response.

Our work adopts the relaxation-homogenization topology optimization method (Cheng and Olhoff, 1982; Murat and Tartar, 1985; Kohn and Strang, 1986; Lurie and Cherkaev, 1986; Bendsøe and Kikuchi, 1988) to design the microstructure field for optimal macroscopic dynamic responses. At each material point, the laminate parameters for a sequentially ranked laminate are assigned and explicit homogenization formulae (Allaire, 2002; Murat and Tartar, 1997) are used to obtain the homogenized material properties. These homogenized properties, together with the trivially computed homogenized mass density, are used in an explicit finite element analysis to compute the structure's macroscopic dynamic response. An analytical sensitivity analysis follows to evaluate the cost and constraint function gradients with respect to the laminate parameters. Finally, an optimality criteria algorithm (refer to Bendsøe and Sigmund, 2003, pp. 9–10 and bibliographical note [7]) updates the laminate parameters, and the process is repeated until convergence is attained.

The remainder of the paper is organized as follows. Section 2 reviews homogenization theory and its application in topology optimization. The transient dynamic optimization problem is defined in Section 3, and the sensitivity analysis is detailed in Section 4. Numerical examples and conclusions are provided in Sections 5 and 6.

2. Homogenization

Most materials, such as steel, are not homogeneous on the microscopic scale, i.e. they contain heterogeneous microstructures. Nonetheless, since we are seldom interested in the microscopic behavior, and we treat these media as homogeneous with effective constitutive properties. Indeed, such effective properties are sufficient for computing most macroscopic responses of interest, e.g. energy and natural frequency.

Effective properties are often obtained by conducting experimental tests on representative samples. However, there are situations, e.g. in composite material design, where we know the constitutive properties of each constituent. In these