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Cavitation in elastomeric solids: II—Onset-of-cavitation surfaces for Neo-Hookean materials

Oscar Lopez-Pamies [a,*], Toshio Nakamura [a], Martín I. Idiart [b,c]

[a] Department of Mechanical Engineering, State University of New York, Stony Brook, NY 11794-2300, USA

[b] Departamento de Aeronáutica, Facultad de Ingeniería, Universidad Nacional de La Plata, Calles 1 y 47, La Plata B1900TAG, Argentina

[c] Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Avda. Rivadavia 1917, Cdad. de Buenos Aires C1033AAJ, Argentina

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ABSTRACT

In Part I of this work we derived a fairly general theory of cavitation in elastomeric solids based on the sudden growth of pre-existing defects. In this article, the theory is used to determine onset-of-cavitation surfaces for Neo-Hookean solids where the defects are isotropically distributed and vacuous. These surfaces correspond to the set of all critical Cauchy stress states at which cavitation ensues; general three-dimensional loadings are considered. Their computation requires the numerical solution of a nonlinear first-order partial differential equation in two variables. The theoretical results indicate that cavitation occurs only for stress states where the three principal Cauchy stresses are tensile, and that the required hydrostatic tensile component increases with increasing shear components. These results are confronted to finite-element simulations for the growth of a small spherical cavity in a Neo-Hookean block under multi-axial loading. Good agreement is found for a wide range of loading conditions. Comparisons with earlier results available in the literature are also provided and discussed. We conclude this work by devising a *closed-form* approximation to the theoretical surface, which is of remarkable accuracy and mathematical simplicity.

1. Introduction

In the preceding paper Lopez-Pamies et al. (2011), henceforth referred to as Part I, we proposed a new strategy to study cavitation in elastomeric solids. The basic idea was to first cast this phenomenon as the homogenization problem of nonlinear elastic materials containing random distributions of *defects*, which were modeled as disconnected nonlinear elastic cavities of zero volume, but of arbitrary shape otherwise. By means of a novel iterated homogenization technique, we then constructed solutions for a specific but fairly general class of distributions and shapes of defects. These included solutions for the change in volume fraction of the underlying defects as a function of the applied loading conditions, from which the onset of cavitation — corresponding to the event when the initially infinitesimal volume fraction of defects suddenly grows into finite values — could be determined. The distinctive features of the theory are that it: (i) allows to consider 3D general loading conditions with arbitrary triaxiality, (ii) is applicable to large (including compressible and anisotropic) classes of nonlinear elastic solids, (iii) incorporates direct information on the initial shape, spatial distribution, and mechanical properties of the underlying defects at which cavitation can initiate, and, in spite of accounting for this

* Corresponding author. Tel.: +1 631 632 8249; fax: +1 631 632 8544.
E-mail addresses: oscar.lopez-pamies@sunysb.edu (O. Lopez-Pamies), toshio.nakamura@sunysb.edu (T. Nakamura), martin.idiart@ing.unlp.edu.ar (M.I. Idiart).

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refined information, (iv) is computationally tractable since the relevant analysis reduces to the study of two Hamilton–Jacobi equations in which the initial volume fraction of defects plays the role of ''time'' and the applied load plays the role of ''space''.

The objective of this paper is to make use of the theory developed in Part I for the first time. Motivated by an application of practical interest that at the same time can lead to results that are as explicit as possible, the problem that we consider here is the construction of an onset-of-cavitation criterion for Neo-Hookean materials, in which the underlying defects are *isotropically distributed* and *vacuous*, under general 3D loading conditions. In this regard, it is appropriate to remark that the Neo-Hookean material model describes reasonably well the response of vulcanized natural rubber (Treloar, 1944), as well as that of other commercially utilized elastomers (see, e.g., Lopez-Pamies, 2010), over a wide range of deformations.[1] Moreover, the distribution of defects is expected to indeed be isotropic within elastomers prepared by most typical processes of synthesis/fabrication. On the other hand, the assumption that the defects are vacuous is adopted here mainly for computational simplicity, but also partly for lack of a better experimentally based constitutive prescription. The emphasis of the selected application aims then at shedding light on the effect of load triaxiality on the onset of cavitation in standard rubber. In this connection, it is important to recall that the majority of cavitation analyses available in the literature have restricted attention to pure hydrostatic loading conditions (Horgan and Polignone, 1995; Fond, 2001). Yet, the occurrence of cavitation is expected to depend sensitively on the triaxiality of the applied loading, and not just on the hydrostatic component (Gent and Tompkins, 1969; Chang et al., 1993; Bayraktar et al., 2008). The results to be generated in this paper seek to quantify this dependence.

In addition to the analytical results derived from the theory of Part I, in this work we also generate numerical finite-element (FE) results for the onset of cavitation in Neo-Hookean materials under general loading conditions. More specifically, our strategy is to generate full 3D solutions for the large-deformation response of a Neo-Hookean block that contains an initially small, vacuous, spherical cavity located at its center. The critical loads at which cavitation occurs are then identified as the affine loads externally applied to the block at which the initially small cavity suddenly grows to a much larger size. While this numerical approach seems simple enough, its presentation in the literature is not known to the authors.

In Section 2, for convenience and clarity, we recall the cavitation criterion developed in Part I for the case when the defects at which cavitation can initiate are isotropically distributed and vacuous. Section 3 deals with the further specialization of this criterion to Neo-Hookean materials and presents the main result of the paper: *inside a Neo-Hookean material, cavitation occurs at a material point P whenever along a given loading path the principal Cauchy stresses* t_i ($i=1,2,3$) *at that point first satisfy the condition*

$$8t_1t_2t_3-12\mu(t_1t_2+t_1t_3+t_2t_3)\Psi+18\mu^2(t_1+t_2+t_3)\Psi^2-27\mu^3\Psi^3-8\mu^3=0 \quad \text{with } t_i>0, \tag{1}$$

where μ denotes the shear modulus of the material in its undeformed state, and $\Psi=\Psi(t_2-t_1,t_3-t_1)$ that satisfies $0<\Psi\leq 1$ and is solution of a first-order nonlinear partial differential equation (pde). Section 4 describes the FE calculations. The analytical and numerical onset-of-cavitation surfaces constructed in Sections 3 and 4 are plotted, discussed, and compared with earlier results in Section 5. Finally, we show in Section 6 that the *closed-form* criterion

$$8t_1t_2t_3-12\mu(t_1t_2+t_1t_3+t_2t_3)+18\mu^2(t_1+t_2+t_3)-35\mu^3=0 \quad \text{with } t_i>0 \tag{2}$$

is a remarkably accurate approximation of the exact criterion (1), which in fact can be utilized in lieu of (1) for all practical purposes.

2. Cavitation criterion: the case of isotropically distributed vacuous defects

In Part I, guided by experimental evidence (Gent, 1991), we considered the phenomenon of cavitation in elastomeric solids as the sudden growth of defects present in a nonlinear elastic material in response to sufficiently large applied external loads. In particular, we considered that the pre-existing defects at which cavitation can initiate are nonlinear elastic cavities of zero volume, but of arbitrary shape otherwise, that are randomly distributed throughout the solid. This point of view led to formulating the problem of cavitation as the homogenization problem of nonlinear elastic materials containing zero-volume cavities, which in turn led to the construction of a fairly general — yet computationally tractable — cavitation criterion. For the case of interest here, when the underlying defects are *isotropically distributed* and *vacuous*, the criterion can be written as follows:

> The onset of cavitation in a nonlinear elastic material with stored-energy function $W(\mathbf{F})$ occurs at critical values $\mathbf{F}_{cr}$ of the deformation gradient tensor $\mathbf{F}$ such that
>
> $$\mathbf{F}_{cr}\in\partial\mathcal{Z}[f_\star(\mathbf{F})] \quad \text{and} \quad 0<\|\mathbf{S}_\star(\mathbf{F}_{cr})\|<+\infty, \tag{3}$$

[1] The Neo-Hookean model has the further merit that it is derivable from statistical mechanics (Treloar, 1943).