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Computational homogenization of plastic porous media with two populations of voids

Younis-Khalid Khdir [a,b], Toufik Kanit [a,b], Fahmi Zaïri [a,b,*], Moussa Naït-Abdelaziz [a,b]

[a] Univ Lille Nord de France, F-59000 Lille, France
[b] Université Lille 1 Sciences et Technologies, Laboratoire de Mécanique de Lille (LML), UMR CNRS 8107, F-59650 Villeneuve d'Ascq, France

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ABSTRACT

The macroscopic yield response of random porous media containing two populations of spherical voids is investigated via large volume computations. The computed yield surface is compared to analytical criteria recently developed for the above-mentioned porous media. To overcome the observed discrepancies, the analytical models are modified by introducing additional parameters which are numerically derived. These parameters depend neither on the void volume fraction nor on the void size. Large volume computations are also used to build the yield surface for porous media containing a single population of randomly distributed spherical voids. The results show that, for an identical fraction of porosities, there is no significant difference between a double and a single population of voids. This result implies that the yield response of porous media containing two populations of voids may be also described by analytical criteria developed in the case of a single population.

1. Introduction

The mathematical description of the plastic behavior of porous media containing spherical voids has been widely investigated since the pioneering works of McClintock [1] and, Rice and Tracey [2]. Green [3] proposed a macroscopic yield function for porous media, and later, Gurson [4] derived analytically an upper bound for the macroscopic yield surface of porous media. This yield function was found triaxiality-dependent although the matrix material obeys the von Mises criterion, which assumes the material incompressible. The modeling, which considers an isotropic, incompressible, rigid and perfectly-plastic matrix, is based on the limit analysis approach, performed on a periodic unit cell. This cell consists in a hollow sphere or cylinder subjected to a uniform macroscopic strain rate at its external boundary. To construct accurate macroscopic yield criteria for porous media, intense researches have been carried out to take into consideration the void shape or the matrix plastic anisotropy. The reader can refer to recent background papers on the subject [5,6].

Several experimental works on metallic or polymeric materials have provided insights on the existence of two populations of voids with different sizes [e.g. [7–9]]. To date, a limited number of theoretical or computational works have been devoted to this problem. Some investigations have been performed using finite element (FE) calculations on a periodic unit cell containing one void embedded in a Gurson type matrix [10]. Others [11] have explicitly incorporated a second population of voids in the matrix surrounding the primary void. To our knowledge, only Perrin and Leblond [12], Vincent et al. [13–15], Julien et al. [16] and Shen et al. [17] have developed analytical models devoted to a double population of voids. By considering the two populations existing at two different scales (microscopic and mesoscopic scales), the modeling is based on a two-step homogenization process, i.e. micro/meso and then meso/macro. After several approximations, the derived macroscopic yield functions are found to be similar to the Gurson form, but depend on the respective volume fraction of the two populations.

Recently, three-dimensional computational homogenization studies have been carried out on random porous materials containing multiple voids [18–23]. To our knowledge, there is no three-dimensional computational homogenization of double porous materials containing multiple voids and using sufficiently large volume elements.

The goal of the current paper is to present a computational homogenization study of porous media containing two populations of spherical voids with different sizes. Computational homogenization is used to obtain the macroscopic yield surface for different stress triaxialities of material volume elements including a large number of randomly distributed voids. The computational data are compared with the macroscopic yield criteria found in the exhaustive literature for such porous materials. Modifications

* Corresponding author at: Université Lille 1 Sciences et Technologies, Laboratoire de Mécanique de Lille (LML), UMR CNRS 8107, F-59650 Villeneuve d'Ascq, France. Tel.: +33 328767460; fax: +33 328767301.
E-mail address: fahmi.zairi@polytech-lille.fr (F. Zaïri).

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of the analytical criteria are proposed to overcome the discrepancies observed with the computational data. Based on the large volume computations, this paper brings two main results. First, the parameters involved in the modified analytical models are found independent of the volume fraction and size of porosities. Secondly, it is found that a porous material with two populations of voids with different sizes can be replaced by a porous material including a single population of voids with an equivalent volume fraction of porosities.

The outline of the present contribution is as follows. Section 2 is devoted to a brief review of existing analytical yield criteria. The results of the computational homogenization are presented and discussed in Section 3. Concluding remarks are given in Section 4.

2. A brief survey of existing analytical models

In this section, a brief survey of macroscopic yield criteria for porous materials containing two populations of spherical voids in a von Mises matrix is presented. The two populations of voids cohabit at two different scales: microscopic and mesoscopic scales. As in any yield criteria for porous media, expressed at the macroscopic scale, the yield function relates the matrix yield stress σ_o, the macroscopic von Mises equivalent stress Σ_{eq} and the macroscopic hydrostatic stress Σ_h. The two macroscopic quantities are expressed as

$$\Sigma_{eq}=\sqrt{\frac{3}{2}\mathbf{\Sigma}':\mathbf{\Sigma}'} \quad \text{and} \quad \Sigma_h=\tfrac{1}{3}\,\mathrm{tr}(\mathbf{\Sigma}) \tag{1}$$

where $\mathbf{\Sigma}$ is the applied macroscopic stress tensor and $\mathbf{\Sigma}'$ is its deviator. The double dot denotes the double contracted product.

Another essential quantity appearing in the yield function is the void volume fraction. Following the works of Vincent et al. [13–15] and Shen et al. [17], the volume fraction of the smallest voids at the smallest (microscopic) scale, denoted f_b, and the volume fraction of the largest voids at the upper (mesoscopic) scale, denoted f_e, are given by

$$f_b=\frac{|\omega_b|}{|\Omega-\omega_e|} \quad \text{and} \quad f_e=\frac{|\omega_e|}{|\Omega|} \tag{2}$$

where Ω denotes the volume of the RVE and, ω_b and ω_e denote the volumes occupied by the voids at the smallest and upper scales, respectively.

The mathematical derivation is based on the assumption of a separation between the two scales of the voids. The total volume fraction of voids f is therefore given by

$$f=\frac{|\omega_b|+|\omega_e|}{|\Omega|}=f_e+f_b(1-f_e) \tag{3}$$

Shen et al. [17] recently established two yield criteria for double porous materials by extending the Ponte Castaneda [24] (PC) and Michel–Suquet [25] (MS) models, originally developed for a single population of voids. The authors [17] performed a two-step homogenization: in the first step, to achieve the transition from the smallest scale to the mesoscale, the PC and MS models were employed to represent the porous matrix at the mesoscale. In the second step, the transition to the macroscale is conducted by identifying the macroscopic yield criterion to a criterion of a porous medium consisting in a compressible Green [3] matrix. At the smallest scale, the solid phase is homogeneous, isotropic and obeys to the pressure-independent von Mises criterion. The Vincent et al. [13–15] yield criterion for double porous materials was derived by considering for the micro/meso homogenization a Gurson-type matrix. A comparison between these analytical models can be found in [17]. Julien et al. [16] extended the Vincent et al. [13–15] yield criterion to include at the microscale a viscoplastic solid phase. In the present work, only the closed-form expressions of PC and MS yield criteria for double porous media, proposed by Shen et al. [17], are retained for further comparisons with the computational results. The PC and MS yield criteria are respectively given by the following formula [17]:

$$\frac{1+\frac{2}{3}f_b}{(1-f_b)^2}\frac{\Sigma_{eq}^2}{\sigma_o^2}+\frac{9f_b}{4(1-f_b)^2}\frac{\Sigma_m^2}{\sigma_o^2}+2f_e\cosh\left(\frac{3}{2}\sqrt{\frac{1+\frac{2}{3}f_b}{(1-f_b)^2}}\frac{\Sigma_m}{\sigma_o}\right)-1-f_e^2=0 \tag{4}$$

$$\frac{1+\frac{2}{3}f_b}{(1-f_b)^2}\frac{\Sigma_{eq}^2}{\sigma_o^2}+\frac{9\left(\frac{1-f_b}{\ln f_b}\right)^2}{4(1-f_b)^2}\frac{\Sigma_m^2}{\sigma_o^2}+2f_e\cosh\left(\frac{3}{2}\sqrt{\frac{1+\frac{2}{3}f_b}{(1-f_b)^2}}\frac{\Sigma_m}{\sigma_o}\right)-1-f_e^2=0 \tag{5}$$

When considering the limit case of zero void content at the smallest scale, i.e. $f_b=0$ and $f_e=f$, both Eqs. (4) and (5) reduce

Table 1
Volume fractions f and number of voids n in the investigated microstructures.

#	f_e	f_b	n_e	n_b
1	0.05	0.05	30	234
2	0.05	0.1	30	468
3	0.1	0.05	60	234
4	0.1	0.0	200	0
5	0.15	0.0	200	0

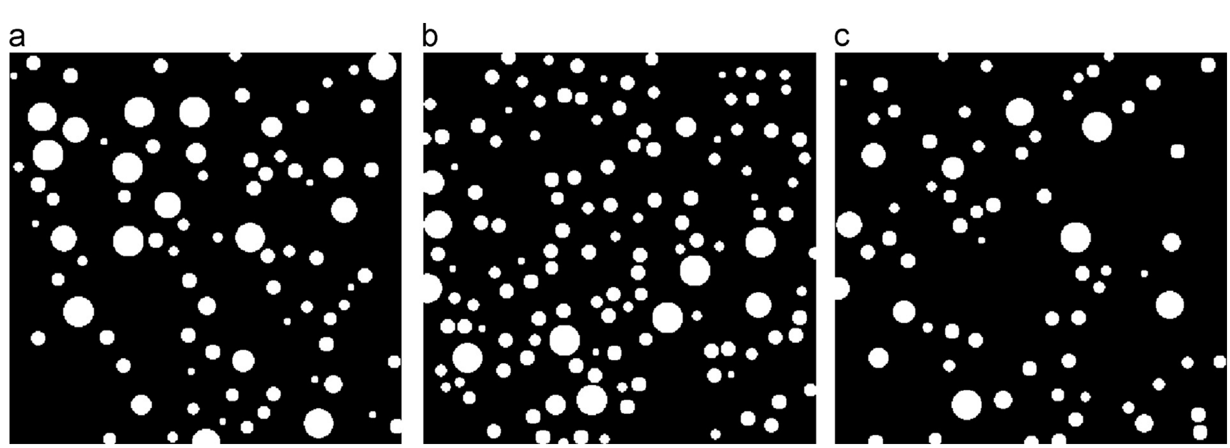

Fig. 1. Cross-sections of investigated double porous microstructures: (a) $f_e=0.1, f_b=0.05$, (b) $f_e=0.05, f_b=0.1$, and (c) $f_e=0.05, f_b=0.05$.