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# Contact laws between solid particles

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ABSTRACT

This paper presents a comprehensive study for the contact laws between solid particles taking into account the effects of plasticity, strain hardening and very large deformation. The study takes advantage of the development of a so-called material point method (MPM) which requires neither remeshing for large deformation problems, nor iterative schemes to satisfy the contact boundary conditions. The numerical results show that the contact law is sensitive to impact velocity and material properties. The contact laws currently used in the discrete element simulations often ignore these factors and are therefore over-simplistic. For spherical particles made of elastic perfectly plastic material, the study shows that the contact law can be fully determined by knowing the relative impact velocity and the ratio between the effective elastic modulus and yield stress. For particles with strain hardening, the study shows that it is difficult to develop an analytical contact law. The same difficulty exists when dealing with particles of irregular shapes or made of heterogeneous materials. The problem can be overcome by using numerical contact laws which can be easily obtained using the material point method.



## 1. Introduction

Understanding the flow behaviour of powders is a key issue in industries such as pharmaceuticals, chemicals, mineral and material processing as well as oil and gas production, agricultural, construction and geo-technical engineering. Over the last decade, the discrete element method (DEM) has emerged as a powerful tool for computer simulations of powder flows. Extensive research has been carried out to develop the method and to apply it to a wide range of problems. The fundamental concept of DEM is rather simple—Newton's second law is integrated for each individual particle in a powder flow. At the core of the DEM is the contact law which describes the relationship between the force and relative displacement between two particles in contact.

### *1.1. The contact law*

The contact law is usually decomposed into a normal and a tangential component. The focus of this paper is on the normal contact because the tangential contact can be considered by the normal contact through a friction coefficient. Most existing contact laws are derived for spherical particles. Fig. 1(a) shows a spherical particle of radius $R$ impacting on a rigid wall, which also represents the symmetry conditions of two identical particles coming into contact. At the beginning of the impact, the force between the particle and the rigid wall, $F$, and the displacement of the centre of the particle, $\delta$, increase forming a circular contact area with radius $a$. Both $F$ and $\delta$ will reach a maximum value but not necessarily at the same

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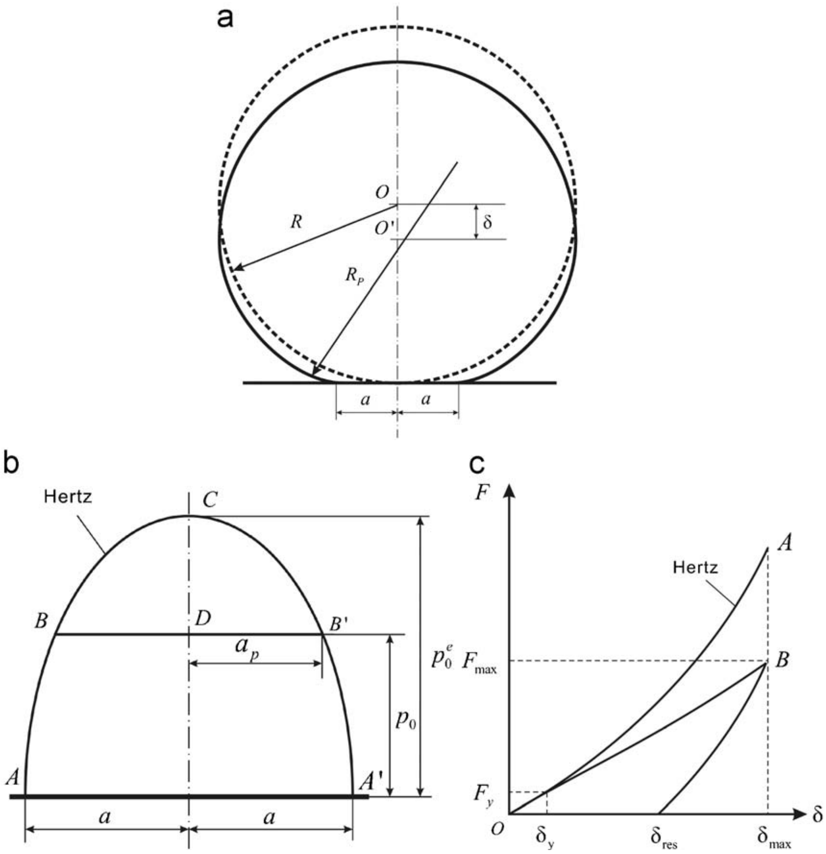


**Fig. 1.** Contact between a sphere and a rigid wall: (a) initial and deformed spheres, (b) pressure distribution within the contact area, and (c) force–displacement relationships.

time. Then the particle rebounds and finally separates from the wall. The $F$–$\delta$ relationship is referred to as the contact law. Although there are various theories addressing the contact law, the basic procedures are mostly the same. On the one hand, the contact force is an integration of the contact pressure distributed over the contact area. This integration provides a connection between the contact force $F$ and the radius of the contact area, $a$. On the other hand, the profile of the sphere near the contact area is represented by a relatively simple function such that a connection is made between $\delta$ and $a$. The contact law is then determined by eliminating the contact radius $a$ from the two connections. In Hertzian theory, the elastic contact pressure distribution $p$ at any point $r$ within the contact area is explicitly given by (e.g. Johnson, 1985)

$$p(r) = p_0^e[1-(r/a)^2]^{1/2} \tag{1}$$

where $p_0^e$ denotes the maximum pressure at the centre of the contact area. Eq. (1) is shown by curve $ACA'$ in Fig. 1(b). The particle shape near the contact area is assumed to be parabolic which can be characterised using a local curvature, $R_p$, as shown in Fig. 1(a) given by

$$R_p = \frac{a^2}{\delta} \tag{2}$$

For small and elastic deformation, Hertz suggested that $R_p = R$. The contact law is then given by

$$F = (4/3)R^{1/2}E^*\delta^{3/2} \tag{3}$$

in which $E^*$ is the effective elastic modulus defined as $E^* = E/(1-\upsilon^2)$ where $E$ and $\upsilon$ are Young's modulus and Poisson's ratio. Eq. (3) is shown by curve $OA$ in Fig. 1(c), for which the loading and unloading curves are identical.

For elastoplastic impact, obtaining the expressions for the contact pressure and the particle profile becomes more difficult. The loading and unloading curves are different due to plastic deformation as shown by curve $OB\delta_{res}$ in Fig. 1(c), where $\delta_{res}$ represents the permanent residual displacement. The particle profile near the contact area keeps changing during the impact due to irreversible piling-up and sinking-in effects. For elastic perfectly plastic spherical particles, Johnson (1985) found that plastic deformation starts beneath the contact area when the maximum contact pressure reaches 1.6 times of the uniaxial yield stress. Johnson (1985) obtained the following relation for the impact velocity, $V_y$,