Available online at www.sciencedirect.com
ScienceDirect
Journal of the Mechanics and Physics of Solids 55 (2007) 1–34
JOURNAL OF THE MECHANICS AND PHYSICS OF SOLIDS
www.elsevier.com/locate/jmps

Structural interfaces in linear elasticity. Part I: Nonlocality and gradient approximations

K. Bertoldi, D. Bigoni*, W.J. Drugan[1]

Dipartimento di Ingegneria Meccanica e Strutturale, Università di Trento, Via Mesiano 77, I-38050 Trento, Italy

Received 9 February 2006; received in revised form 13 June 2006; accepted 15 June 2006

Abstract

Many biological and optimal materials, at multiple scales, consist of what can be idealized as continuous bodies joined by structural interfaces. Mechanical characterization of the microstructure defining the interface can nowadays be accurately done; however, such interfaces are usually analyzed employing models where those properties are overly simplified. To introduce into the analysis the microstructure properties, a new model of structural interfaces is proposed and developed: a true structure is introduced in the transition zone, joining continuous bodies, with geometrical and material properties directly obtained from those of the interfacial microstructure. First, the case of an elliptical inclusion connected by a structural interface to an infinite matrix is solved analytically, showing that nonlocal effects follow directly from the introduction of the structure, related to the inclination of the connecting elements. Second, starting from a discrete structure, a continuous model of a structural interface is derived. The usual zero-thickness linear interface model is shown to be a special case of this more general continuous structural interface model. Then, a gradient approximation of the interface constitutive law is rigorously derived: it is the first example of the analytical derivation of a nonlocal interface model from the microstructure properties. The effects introduced in the mechanical behavior by both the continuous model and its gradient approximation are illustrated by solving, for the first time, the problem of a circular

*Corresponding author. Tel.: +39 461882507; fax: +39 461882599.
E-mail addresses: katia.bertoldi@ing.unitn.it (K. Bertoldi), davide.bigoni@ing.unitn.it, bigoni@ing.unitn.it (D. Bigoni), drugan@engr.wisc.edu (W.J. Drugan).
URL: http://www.ing.unitn.it/~bigoni/.
[1]Permanent address: Department of Engineering Physics, University of Wisconsin-Madison, 1500 Engineering Drive, Madison, WI 53706, USA.

doi:10.1016/j.jmps.2006.06.004

inclusion connected to an infinite matrix by a structural interface and subject to remote uniform stress.

Keywords: Mechanical interfaces; Elliptical inclusions; Structural interfaces; Gradient models; Nonlocal effects

1. Introduction

There are many mechanical problems involving interfaces joining different parts of a continuous body. These transition zones are often characterized by well-defined microstructures: in Pinctada nacre, fibrils of organic matrix bridge the platelet lamellae (detail (A) of Fig. 1); a truss-like structure made of glass fibers bridges a crack in a short glass-fiber-reinforced polypropylene (detail (B) of Fig. 1); craze fibrils bridge the two bulk polymer surfaces at a crack tip in polystyrene (detail (C) of Fig. 1); in the meninges surrounding the human brain, trabeculae connect the subarachnoid space with the pia mater (detail (D) of Fig. 1); in a palm petiole, the relatively dense vascular bundles are distributed throughout a 'web' of parenchyma cells (detail (E) of Fig. 1). Currently, the possibility to produce artificial material with this type of microstructure is being studied:

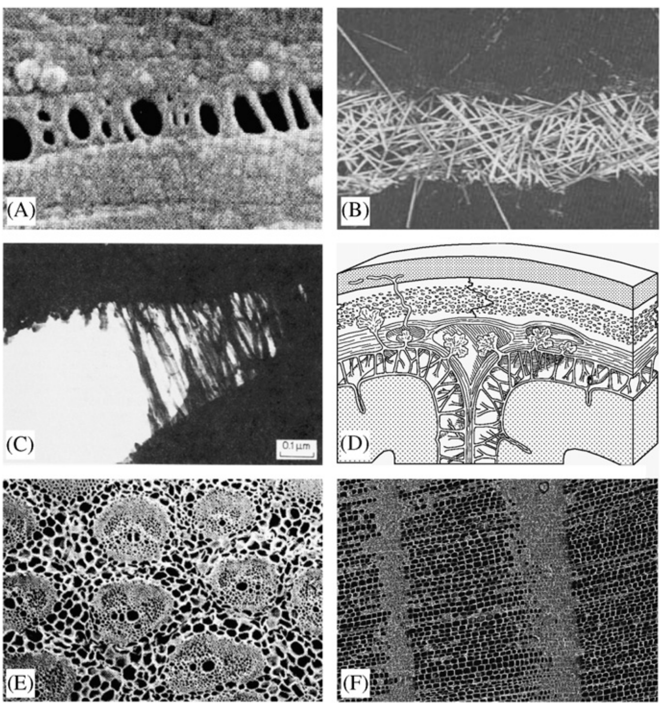

Fig. 1. Examples of structural interfaces in nature: (A) Pinctada nacre (figure from Jackson et al., 1988); (B) short glass-fiber-reinforced polypropylene (taken from Geers, 1997); (C) crack tip in polystyrene (taken from Xiao and Curtin, 1995); (D) meninges surrounding the human brain (adapted from Kahle and Frotscher, 2002); (E) cross-section of a palm (chamaerops humilis) petiole (taken from Gibson et al., 1995); (F) pyrolyzed wood infiltrated with Si (courtesy of Dr. L. Esposito, ISTEC CNR, Faenza, Italy).