Solid State Communications 273 (2018) 34–38

Contents lists available at ScienceDirect

Solid State Communications

journal homepage: www.elsevier.com/locate/ssc

Communication

Geometric phase for a two-level system in photonic band gab crystal

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ARTICLE INFO

Communicated by X.C. Shen

Keywords:
A. Solid state systems
B. Photonic crystals
C. Geometric phase
D. Detuning parameter

ABSTRACT

In this work, we investigate the geometric phase (GP) for a qubit system coupled to its own anisotropic and isotropic photonic band gap (PBG) crystal environment without Born or Markovian approximation. The qubit frequency affects the GP of the qubit directly through the effect of the PBG environment. The results show the deviation of the GP depends on the detuning parameter and this deviation will be large for relatively large detuning of atom frequency inside the gap with respect to the photonic band edge. Whereas for detunings outside the gap, the GP of the qubit changes abruptly to zero, exhibiting collapse phenomenon of the GP. Moreover, we find that the GP in the isotropic PBG photonic crystal is more robust than that in the anisotropic PBG under the same condition. Finally, we explore the relationship between the variation of the GP and population in terms of the physical parameters.

The concept of geometric phase in quantum system was originally introduced by Berry [1] when he studied the dynamics of a closed quantum system which undergoes an adiabatic cyclic evolution. By relaxing the superfluous assumptions, such as periodicity and adiabatic evolution, geometric phase was generalized to a much wider setting: for a cyclic but non-adiabatic evolution, Aharonov and Anandan proved the existence of a Hamiltonian independent phase, which is called *AA* phase [2]. The classical counterpart, Pancharatnam phase, leads to the generalization of geometric phase for almost arbitrary unitary evolution [3,4]. The geometric phase has been observed experimentally in optical [5], NMR [6,7], and superconducting electronic circuit experiments [8,9]. When the initial state is orthogonal to the final state, the definition of geometric phase breaks down, and Manini et al. introduced a complementary concept called off-diagonal geometric phase [10] to recover the phase information, which was verified by Hasegawa et al. in the neutron interference experiment [11]. Recently, the renewed interest in the investigation of GP comes from the application of the GP to implement the logic gates in quantum computation [12]. The purely geometric nature of the phase makes such computation intrinsically fault-tolerant and robust against certain types of classical fluctuation noise [13–17].

The eventuality of preservation is extremely dependent on the intrinsic properties of the environment that acts on the quantum system. All realistic quantum systems can never be isolated from the surrounding environment completely. Therefore, in most practical cases the interaction of a given system with its environments cannot be neglected and act as sources of decoherence and dissipative which makes to use the theory of open quantum systems [18]. Thus, it is an important subject to analyze the quantum decay induced by the unavoidable interaction with the environment. For this reason, the study of open quantum systems taking into account the effect of the environment on the dynamical evolution of the system of interest has attracted much attention in the investigations of modern quantum theory, particularly of quantum-information processing.

On the other hand, in photonic crystals, the periodicity structures lead to the formation of a PBG [19]. The presence of the PBG in the dispersion relation of the electromagnetic field results in a series of phenomena, including the inhibition of the spontaneous emission [20], strong localization of light [21], and formation of atom-photon bound states [22]. The GP of a quantum system is a potential resource for quantum informatics by means of the holonomic quantum computing. No realistic physical quantum system is in perfect isolation from its environment, which is essentially present in each laboratory clearly, in general, spoils the phase. Robustness of the GP with respect to the environmental effects is a basic condition for an effective quantum computation. Here, we have examined in detail the GP of a qubit system coupled to its own anisotropic and isotropic photonic band gap (PBG) crystal environment without Born or Markovian approximation. We consider the case of anisotropic and isotropic dispersion of the PBG crystal and show that the GP is strictly depends on the detuning param-

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https://doi.org/10.1016/j.ssc.2017.11.013
Received 13 May 2017; Accepted 24 November 2017
Available online 1 February 2018
0038-1098/

eter and it can be considered as a tool for testing and characterizing the nature effect between the qubit and PBG crystal interaction. Finally, we highlight the connection between the variation of the GP and two-level atom excited-state population in terms of the physical parameters.

The system we investigate is a two-level atom coupled to the radiation in a photonic crystal with isotropic and anisotropic models. In the rotating-wave approximation, the Hamiltonian for the coupled atom-field system can be written as

$$H = \hbar\omega_{12}\sigma_{22} + \sum_k \omega_k a_k^\dagger a_k + i\hbar\sum_k g_k\left(a_k^\dagger\sigma_{12} - \sigma_{21}a_k\right), \tag{1}$$

where $\sigma_{ij} = |i\rangle\langle j|$ $(i,j = 1,2)$ are the atomic operators for a two-level atom with excited state $|2\rangle$, ground state $|1\rangle$, and resonant transition frequency ω_{12}. a_k and $a_k^\dagger$ are the annihilation and creation operators of the radiation field. ω_k is the radiation frequency of mode k in the reservoir, and the atom-field coupling constant $g_k = \left(\omega_{12}d_{12}/\hbar\right)\sqrt{\left[\hbar/\left(2\epsilon_0\omega_k V\right)\right]}\hat{e}_k.\hat{u}_d$ is assumed to be independent of atomic position with the fixed atomic dipole moment $\vec{d}_{12} = d_{12}\hat{u}_d$. V is the sample volume, $\hat{e}_k$ is the polarization unit vector of the reservoir of mode k, and the Coulomb constant is ϵ_0. In the single photon sector, the wave functions of the system have the form

$$\psi_1 = |2\rangle\otimes\prod_k|0_k\rangle,$$

$$\psi_k = |1\rangle\otimes|1_k\rangle\prod_{k'\neq k}|0_{k'}\rangle, \tag{2}$$

where the ket $|0_k\rangle$ indicates that the field mode k is a vacuum state. $|1_k\rangle$ denotes that the field mode k is in the first excited state. The unexcited state

$$\psi_0 = |1\rangle\otimes\prod_k|0_k\rangle \tag{3}$$

is not coupled to any other state.

The general state vector could be written as

$$\psi(t) = q_0\psi_0 + q_1e^{-i\omega_{21}t}\psi_1 + \prod_k q_ke^{-i\omega_kt}\psi_k \tag{4}$$

in terms of the states (2) and (3) and insert this into the Schrödinger to obtain the following set of coupled equations:

$$\dot{q}_1 = -i\sum_k g_ke^{-i\Omega_kt}q_k \tag{5}$$

$$\dot{q}_k = -ig_ke^{-i\Omega_kt}q_1, \tag{6}$$

with detuning frequency $\Omega_k = \omega_k - \omega_{12}$. By integrating (6) with the initial condition $q_k(0)$ and substituting the result into (5), we take the time involving equation of the excited-state probability amplitude

$$\dot{q}_1 = -\int_0^\infty d\tau G(t-\tau)q_1(\tau) \tag{7}$$

with the memory Kernel $G(t-\tau) = \sum_k g_k^2e^{-i\Omega_k(t-\tau)}$ which presents the delay Green's function of the problem. Thus, we could obtain the atomic reduced density matrix as

$$\widehat{\rho}(t) = \begin{pmatrix}\rho_{22}(0)|q(t)|^2 & \rho_{21}(0)q(t)\\ \rho_{12}(0)q^*(t) & \rho_{11}(0)+\rho_{22}(0)\left(1-|q(t)|^2\right)\end{pmatrix}. \tag{8}$$

The atomic state dynamics is depends only on the function $q(t)$, whose explicit time dependence contains the information about the environment spectral density and coupling constant. This shows that the form of the GP does not explicitly depend on the particular choice of the environment but only on the Hamiltonian model of (1) and on the chosen of initial states.

To calculate the GP for the qubit undergoing non-unitary evolution, we use the gauge invariant expression derived in Ref. [23]

$$\Phi = \arg\left(\sum_k\sqrt{\chi_k(0)\chi_k(\tau)}\tau\langle\omega_k(0)|\omega_k(\tau)\rangle e^{-\int_0^\tau dt\langle\omega_k(t)|\dot{\omega}_k(t)\rangle}\right), \tag{9}$$

where $\chi_k(t)$ and $|\omega_k(t)\rangle$ are the eigenvalues and corresponding eigenkets of the reduced density matrix $\rho(t)$. One may view the GP factor defined in Eq. (9) as a weighted sum over the phase factors pertaining to the eigenkets of the reduced density matrix. Thus the detail of expression for the geometric phase would depend on the digitalization of the reduced density matrix (8).

The object of our study, the evaluate the GP, could be obtained by solving (9). Here we consider as environment the case of anisotropic and isotropic PBG crystals with dispersion relation ω_k. In an anisotropic dispersion model, appropriate to fabricated PBG materials, we associate the band edge with specific point in k-space, $\mathbf{k}-\mathbf{k}_0$. By preserving the vector character of the dispersion expended about $\mathbf{k}_0$, both direction and magnitude of the band edge wave-vector are modified. This gives a dispersion relation of the form [24]

$$\omega_{\mathbf{k}} \approx \omega_c + A\left(\mathbf{k}-\mathbf{k}_0\right)^2, \tag{10}$$

where A is a model-dependent constant, ω_c is the upper band-edge frequency and $\mathbf{k}_0$ is a specific wave vector related to the point-group symmetry of the dielectric material. $\delta = \omega_{12} - \omega_c$ is the detuning of the atomic frequency with respect to the band-edge frequency and $\alpha \approx \omega_{12}^2d^2/[8\omega_c\epsilon_0(\pi A)^{3/2}]$ is a constant that depends on the nature of the band-edge singularity. Using the Laplace transform, we get

$$q(t) = \varepsilon\left\{\lambda_+e^{i\lambda_+^2t}\left[1+\Phi\left(\lambda_+e^{i\pi/4}\sqrt{t}\right)\right] - \lambda_-e^{i\lambda_-^2t}\left[1+\Phi\left(\lambda_-e^{i\pi/4}\sqrt{t}\right)\right]\right\}, \tag{11}$$

where

$$\varepsilon = \frac{e^{i\delta t}}{\sqrt{\alpha^2-4\delta}},$$

$$\lambda_+ = (-\alpha+\sqrt{\alpha^2-4\delta})/2,$$

$$\lambda_- = \left(-\alpha-\sqrt{\alpha^2-4\delta}\right)/2,$$

and Φ is the error function. The parameter α^2 scales the time evolution and for large times (α^2t) one has $q(t)\to$ const., that is, an asymptotic population trapping.

Let us now consider the case of an environment as zero-temperature three-dimensional periodic dielectric with isotropic photon dispersion relation ω_k [25,26]. The dispersion relation near the band edge ω_c can be given by

$$\omega_k = \omega_c + D(k-\overline{k})^2, \tag{12}$$

where $D \cong \omega_c/\overline{k^2}$. This dispersion relation is isotropic since it depends only on the magnitude k of the wave vector $\mathbf{k}$. In that case, the explicit form of $q(t)$ can be obtained as

$$q(t) = 2a_1x_1e^{\beta x_1^2t+i\delta t} + a_2(x_2+y_2)e^{\beta x_2^2t+i\delta t} - \sum_{j=1}^3 a_jy_j\left[1-\Phi\left(\sqrt{\beta x_j^2t}\right)\right]e^{\beta x_j^2t+i\delta t}, \tag{13}$$

where

$$x_1 = \left(A_+ + A_-\right)e^{i\pi/4} \tag{14}$$

$$x_2 = \left(A_+e^{-i\pi/6} - A_-e^{i\pi/6}\right)e^{-i\pi/4} \tag{15}$$

$$x_3 = \left(A_+e^{i\pi/6} - A_-e^{-i\pi/6}\right)e^{i3\pi/4}, \tag{16}$$