Solid State Communications 247 (2016) 12–16

Contents lists available at ScienceDirect

Solid State Communications

journal homepage: www.elsevier.com/locate/ssc

Communication

Dicke-Josephson effect in a cross-typed triple-quantum-dot junction

Xiao-Qi Wang, Guang-Yu Yi, Wei-Jiang Gong*

[a] College of Sciences, Northeastern University, Shenyang 110819, China

ARTICLE INFO

Article history:
Received 9 March 2016
Received in revised form
9 July 2016
Accepted 11 August 2016
Available online 12 August 2016

Keywords:
A. Quantum dot
B. Dicke effect
D. Josephson current

ABSTRACT

We investigate the Dicke-Josephson effect in a superconductor/triple-quantum-dot/superconductor junction in which the central dot is coupled to the superconductors. It is found that the Dicke effect can modulate the Josephson effect in a nontrivial way. In the noninteracting case, the Dicke effect induces a subpeak in the supercurrent spectrum around the energy zero point. When intradot interactions are taken into account, the role of the Dicke effect changes completely. Namely, it tends to suppress the π-phase current near the position of electron-hole symmetry. With the increase of the Coulomb strength, it has an opportunity to reverse the current direction. We thus conclude that the Dicke-Josephson effect is also an important part in describing the Josephson effect in coupled-dot junctions.

1. Introduction

The Dicke effect, one concept from the quantum optics, describes the presence of a strong and narrow spontaneous emission line in addition to much broader lines of a collection of atoms, which are separated by a distance smaller than the wavelength of the emitted light [1,2]. Following the development of low-dimensional physics, the Dicke effect in the mesoscopic system has attracted much attention. It has first been predicted in two-channel resonant tunneling [3], since then the analogies to the Dicke effect have been found in some other systems [4–10]. In a coupled double-quantum-dot system under a magnetic field, the Dicke effect can be controlled by the magnetic flux [5]. The properties of transmission spectrum and the local density of states in a quantum dot (QD) side-coupled to a quantum wire can also be attributed to the Dicke effect [6,7]. More recently, the Dicke effect has been predicted in a QD coupled to two side QDs in the Coulomb blockade regime [11] and Kondo regime [12,13], in which the effective coupling between localized levels and a conduction channel gives rise to effectively fast supertunneling and slow subtunneling modes. Since the local density of state can exhibits almost a δ-like shape for appropriate parameters due to the Dicke effect, one expects a significant enhancement of the thermal efficiency [14].

Alternative important transport through mesoscopic systems is the well-known Josephson effect, in the case of superconductors coupling to the central device [15,16]. It has been reported that in such Josephson junctions, interesting competition between two many-body correlations, i.e., the electron correlation effect in the QD and the Copper-pair correlation, has been observed [17]. For instance, the Coulomb repulsion in a QD enables the electrons in a Cooper pair to tunnel one by one via virtual processes in which the spin ordering of the pair is reversed, leading to the π-junction behavior. If the Kondo temperature far exceeds the superconducting gap, the induced Kondo resonance level will restore the 0-junction state of the supercurrent. Accordingly, one can drive a phase transition between spin singlet (0 junction) and doublet (π junction) states by controlling the competition between correlations [18,19]. Compared with a single-QD Josephson junction, the Josephson phase transitions present new and interesting properties in the junctions with QD molecules [20–23]. When a double-QD molecule is embedded in the Josephson junction, the current phase can be varied by the adjustment of the interdot coupling [21,22]. Furthermore, the complication of QD-molecule geometry induces the quantum-interference effects which inevitably adjust the phase transition behaviors. For instance, in a T-shaped double-QD structure, a novel $0-\pi$ transition behavior can be observed in the half-filled case, which does not occur in the serially-coupled double-QD system [24]. In the Fano-Josephson junction, an intermediate bistable phase is found to appear in the $0-\pi$ phase-transition process, with the suppression of the Josephson current [25,26].

Since the quantum interference is an important factor to adjust the Josephson effect, it can be anticipated that the Dicke effect can modify the Josphson effect in a nontrivial way. With this idea, we investigate the Josephson effect in a superconductor/triple-quantum-dot/superconductor (S/TQD/S) junction in which the central QD is coupled to the superconductors. It is found that in the noninteracting case, the Dicke effect induces a subpeak in the supercurrent spectrum around the energy zero point. When

---

* Corresponding author.
E-mail address: gwj@mail.neu.edu.cn (W.-J. Gong).

http://dx.doi.org/10.1016/j.ssc.2016.08.011
0038-1098/

intradot interactions are taken into account, it suppresses the π-phase current near the position of electron-hole symmetry. Moreover, it can reverse the current direction with the further increase of Coulomb strength. Thus, it is certain that the Dicke effect can be an important factor to modify the Josephson effect in coupled-QD junctions.

2. Theory

The Hamiltonian of the S/TQD/S structure is given by $H = \sum_\alpha H_\alpha + H_D + H_T$ where

$$H_\alpha = \sum_{k\sigma} \varepsilon_{\alpha k} a^\dagger_{\alpha k\sigma} a_{\alpha k\sigma} + \sum_k \left(\Delta e^{i\varphi_\alpha} a_{\alpha k\downarrow} a_{\alpha -k\uparrow} + \Delta e^{-i\varphi_\alpha} a^\dagger_{\alpha -k\uparrow} a^\dagger_{\alpha k\downarrow} \right), H_D \\ = \sum_{\sigma,j=1}^{3} \varepsilon_j d^\dagger_{j\sigma} d_{j\sigma} + \left(t_1 d^\dagger_1 d_{2\sigma} + t_2 d^\dagger_1 d_{3\sigma} + h.\,c. \right) + \sum_j U_j n_{j\uparrow} n_{j\downarrow}, H_T \\ = \sum_{\alpha k\sigma} \left(V_{\alpha k} a^\dagger_{\alpha k\sigma} d_{1\sigma} + h.\,c. \right). \tag{1}$$

$H_\alpha(\alpha = L, R)$ is the standard BCS mean-field Hamiltonian for the superconductors with phase φ_α and energy gap Δ. H_D models the TQD, and H_T denotes the tunneling between superconductor-$L(R)$ and QD-1. $a^\dagger_{\alpha k\sigma}$ and $d^\dagger_{j\sigma}$ ($a_{\alpha k\sigma}$ and $d_{j\sigma}$) are operators to create (annihilate) an electron with momentum k and spin σ in lead-α and in QD-j, respectively. $\varepsilon_{\alpha k}$ and ε_j denote the corresponding energy levels. U_j indicates the strength of intradot Coulomb repulsion, and t_j is the interdot coupling coefficient. $V_{\alpha k}$ denotes the coupling between lead-α and QD-1.

In such a structure, the Josephson current at zero temperature can be evaluated by deriving the ground-state (GS) energy E_{GS} with respect to the superconducting phase difference, i.e.,

$$I_J = \frac{2e}{\hbar} \frac{\partial E_{GS}(\varphi)}{\partial \varphi}, \tag{2}$$

where $\varphi = \varphi_L - \varphi_R$. However, due to the presence of continuum state in this system, the determination of the ground state is a formidable challenge. It is thus necessary to introduce some approximation scheme, in which a great simplification should be included by integrating out the electronic degrees of freedom of the superconductors. It is known that one feasible approach, usually referred to as the zero bandwidth model (ZBWM), has been discussed in some previous studies. [27] This approach leads to an effective low energy theory in which each superconductor is replaced by a single site with an effective pairing potential $\tilde{\Delta}$. Also, the hopping term $\tilde{V}_\alpha$ is replaced by an effective parameter $\tilde{V}_\alpha$. Accordingly, the new expressions of H_α and H_T are given by

$$H_\alpha = \sum_\sigma \varepsilon_\alpha a^\dagger_{\alpha\sigma} a_{\alpha\sigma} + \tilde{\Delta} e^{i\varphi_\alpha} a_{\alpha\downarrow} a_{\alpha\uparrow} + \tilde{\Delta} e^{-i\varphi_\alpha} a^\dagger_{\alpha\uparrow} a^\dagger_{\alpha\downarrow}, H_T \\ = \sum_\sigma \left(\tilde{V}_L a^\dagger_{L\sigma} d_{1\sigma} + \tilde{V}_R a^\dagger_{R\sigma} d_{2\sigma} + h.\,c. \right). \tag{3}$$

It has been reported that the ZBWM can give qualitatively correct results and can grasp the ground state properties in this kind of systems in the approximate range $\Gamma \le \Delta$, where Γ is the standard tunneling rate to the leads [24].

It can be readily found that for our considered structure, the Hilbert space of the new system within the ZBWM is restricted to 4^5 states and the z component of the total spin S is a good quantum number. Thus, the eigenstates can be characterized in terms of S_z and the eigenenergies can be obtained by the block diagonalization of the Hamiltonian matrix. In addition, as for the Josephson current, it is characterized by its phase. To be concrete, if the GS energy as a function of φ has a global minimum at the point $\varphi = 0$ ($\varphi = \pi$), the current will be located as its $0(\pi)$ phase. For the $0'(\pi')$ phase, it describes the case where one local minimum emerges at the point $\varphi = \pi(\varphi = 0)$ in the E_{GS} spectrum, in addition to the global minimum at point $\varphi = 0(\varphi = \pi)$ [28,29].

3. Numerical results and discussions

With the help of the theory developed above, we next calculate the Josephson current in the S/TQD/S structure. With respect to the structural parameters, we take them to be the following values: $\tilde{\Delta} = \tilde{V}_\alpha = 1.0$ and $t_j = t_0 = 2.0$. Additionally, in order to clarify the influence of the Dicke effect on the Josephson-current property, we choose the QD levels to be $\varepsilon_1 = \varepsilon_0$, $\varepsilon_2 = \varepsilon_0 + \delta$, and $\varepsilon_3 = \varepsilon_0 - \delta$ (Fig. 1).

Firstly, we would like to investigate the influence of the Dicke effect on the Josephson current in the noninteracting case. The numerical results are shown in Fig. 2. As a typical case, the Josephson phase difference is taken to be $\varphi = \frac{\pi}{2}$. In this figure, one can find that in the case of $\delta = 0$, two wide current peaks appear in the supercurrent spectra, and that their positions are near the points of $\varepsilon_0 \approx \pm 2\sqrt{2}$, respectively. The reason for this result can be explained as follows. The dc Josephson current, driven by the Josephson phase difference, is related to the eigenlevels of the QD molecule. For a noninteracting QD molecule with $\varepsilon_j = \varepsilon_0$ and $t_j = t_0$, its eigenlevels are $E_1 = \varepsilon_0 - \sqrt{2}\,t_0$, $E_2 = \varepsilon_0$, and $E_3 = \varepsilon_0 + \sqrt{2}\,t_0$, with the corresponding molecule states $\psi_1 = \left[\frac{1}{2}, \frac{-1}{\sqrt{2}}, \frac{1}{2}\right]^T$, $\psi_2 = \left[\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right]^T$, and $\psi_3 = \left[\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right]^T$. Since the central QD couples to the superconductors, the first and third molecule states contribute to the Josephson current, leading to the appearance of two peaks in the supercurrent spectra.

When a nonzero δ is taken into account to destroy the level symmetry among the QDs, one subpeak begins to appear around the energy zero point in the supercurrent spectrum, as shown in Fig. 2. Besides, its magnitude is continually enhanced with the increase of δ. As typical cases, in the case of $\delta = 0.5$, the subpeak value is about equal to 0.12, and its value increases to 0.12 when $\delta = 1.0$. Surely, the appearance of such a subpeak originates from the Dicke effect, analogous to the conductance result in the case of normal metallic leads. Its reason be attributed to the change of the transmission paths because of the molecular eigenlevel shift by the presence of δ. Via a simple calculation, we can find that in the presence of δ, the eigenlevels are: $E_1 = \varepsilon_0 - \sqrt{\delta^2 + 2t_0^2}$, $E_2 = \varepsilon_0$, and $E_3 = \varepsilon_0 + \sqrt{\delta^2 + 2t_0^2}$, with $\psi_2 = \left[\frac{t}{\sqrt{\delta^2 + 2t_0^2}}, \frac{\delta}{\sqrt{\delta^2 + 2t_0^2}}, \frac{-t}{\sqrt{\delta^2 + 2t_0^2}}\right]^T$. It clearly shows that a nonzero δ causes ψ_{22} to not be equal to zero. This exactly introduces a new paths for the Cooper-pair tunneling and then leads to the appearance of one subpeak around the energy point (related to E_2) in the supercurrent spectrum. For simplicity, we call this phenomenon the Dicke-Josephson effect. However, it can be found that different from the electron-transmission result, the Dicke effect only induces a subpeak around the energy zero

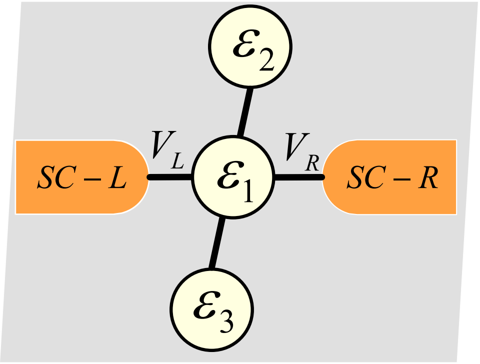

Fig. 1. Schematic of a S/TQD/S structure. QD-1 is connected to two s-wave superconductors and QD-2(3) is side coupled to QD-1.