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Thermal quantum Fisher information in quantum dot system

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ABSTRACT

Using the quantum Fisher information (QFI), we investigate the problem of the parameter estimation in quantum system dot (QDS) including the effects of different parameters. We find that the QFI is affected by the strength temperature and might be finite even for higher temperatures in the asymptotic limit. Furthermore, we show that there is an optimal value of temperature such that the precision of the parameter estimation is maximal and that revivals and retardation of information loss may occur by adjusting the initial conditions. Finally, we show that this quantity may be proposed to detect the amount of the total quantum information that a QDS state contains with respect to projective measurements.

1. Introduction

QFI, which detects the sensitivity of the state variation with respect to changes in a parameter estimation, is one of the most quantity for both quantum estimation theory and quantum information theory. Parameter estimation is a significant pillar of different branches of science and technology, and developed new techniques in measurement for parameter sensitivity that has often led to scientific breakthroughs and technological advancement. In the field of quantum estimation, the main task is to determine the value of an unknown parameter labeling the quantum system, and primary goals are to enhance the precision of resolution. There is a great deal of work on phase estimation addressing the practical problems of state generation, loss, and decoherence [1–6]. Fisher information lies at the heart of a parameter estimation theory that was originally introduced by Fisher [7]. QFI, which characterizes the sensitivity of the state with respect to changes in a parameter, is a key concept in parameter estimation theory. It provides in particular a bound to distinguish the members of a family of probability distributions. When quantum systems are involved, especially for problems in which the quantity of interest is not directly accessible, the optimal measurement may be found using tools from quantum estimation theory. The quantum version of the Cramér–Rao inequality has been established and the lower bound is imposed by QFI [8]. Hence, the QFI becomes the key problem to be solved. An abstract quantity that measures the maximum information about a parameter ϕ that can be extracted from a given measurement procedure.

Actually, an important goal in solid-state quantum physics is to enhance the amount of the resolution. The motivation behind this quest comes both from the fact that parameter estimation for electrons in a solid-state structure has not yet been proved and from the recent experimental progress in the field of quantum information processing in these systems, which has, among other things, led to experimental realization of single and two-qubit manipulations of electron spin qubits in quantum dots [9–11] and coherent control of spins in diamond [12]. Many aspects of these quantum systems, such as hyperfine coupling to the nuclear spins [13,14], the spin blockade [15,16], implementation of the singlet–triplet qubit by confining two electrons in QD systems in a two-dimensional electron gas (2DEG) located below the surface of a GaAs–AlGaAs heterostructure [17], and the effects of applying a slanting magnetic field [18] are currently active topics of research. Here, we will address this problem by calculating the QFI in an isolated quantum dot, electron–electron interaction at the mean field level, in terms of different parameters of the QDS involved in the thermal state for different ranges of the temperatures. Such a system can be employed to perform logical operations, which can be used to implement a universal quantum information and computation. Furthermore, we show that the QFI may be proposed to measure the amount of the total quantum correlation that a QDS state contains with respect to projective measurements.

This paper is structured as follows. In Section 2, we present a review of the QFI and define the different steps of the interferometric scheme. Furthermore, we give the methodology for studying

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the precision of the parameter estimation in the QDS. In Section 3, we present the model for the QDS and describe the dependence on different input parameters. In Section 4, we present the related major results with discussion. Section 5 is devoted to the comparison between the QFI and the quantum discord in QDS. We conclude our work in Section 6.

2. Quantum Fisher information

We first present a brief review of the QFI. A crucial goal of quantum estimation is to archive the best observable. For example, in order to estimate the true value of parameter θ provided that the system is in one state of the family $\{\rho_\theta\}$, an observable $\hat{\theta}$ is said to be the unbiased estimator, that is, the expectation of the estimator should satisfy $\mathrm{Tr}(\rho_\theta\hat{\theta})=\theta$ and in general the estimator $\hat{\theta}$ is not unique. We can quantify how accurately a state can measure an unknown parameter with the QFI associated with quantum Cramér–Rao (QCR). QFI is defined as

$$F_Q=\mathrm{Tr}[\rho_\theta L^2], \tag{1}$$

where ρ_θ is the density matrix of the system, θ is the parameter to be measured, and L is the symmetric logarithmic derivation given by

$$\frac{\partial\rho_\theta}{\partial\theta}=\frac{1}{2}[L\rho_\theta+\rho_\theta L], \tag{2}$$

where the QFI does not depend on the particular choice of $L\rho_\theta$.

The QCR inequality has been formulated, in which the bound is asymptotically archived by the maximum likelihood estimator as well as the classical theory,

$$\Delta\theta\geq(\Delta\theta_{\mathrm{QCR}})=\frac{1}{\sqrt{F_Q}}\geq\frac{1}{\sqrt{L^2}}, \tag{3}$$

where $(\Delta\theta)^2$ is the mean square error in the parameter θ. The above inequality defines the principally smallest possible uncertainty in the value of the parameter estimation. The measurement uncertainty $\Delta\theta$ is quantified through the units corrected, root-mean deviation of the estimate parameter θ from its true value

$$\Delta\theta=\frac{\theta_{\mathrm{est}}}{|d\langle\theta_{\mathrm{est}}\rangle_\theta/d\theta|}-\theta. \tag{4}$$

Consider in general a readout on the probe described by a POVM with one parameter family of the elements $E(\xi)$

$$\int d\xi E(\xi)=\mathbf{1}. \tag{5}$$

Let $p(\xi|\theta)=\mathrm{tr}(E(\xi)\rho_\theta)$ be the measured probabilities from the various outcomes of the POVM when the true value of the measured parameter is θ. The QFI is given by [19,20]

$$F_Q=\max_{\{E(\xi)\}}F \tag{6}$$

where F is the classical Fisher information computed by the probability distribution for the measurement outcomes

$$F=\int d\xi\ \mathrm{tr}(E(\xi)L^2\rho_\theta)=\mathrm{tr}(L^2\rho_\theta)=\langle L^2\rangle. \tag{7}$$

The maximization in Eq. (6) is over all possible readout procedures (POVMs) on the probe. Such a maximization is indeed a daunting task and even if it can be done, implementing the POVM that maximizes the Fisher information, thereby minimizing the measurement uncertainty, may, in all likelihood, be impossible to implement in the laboratory. The classical Fisher verified the inequality [19,20]

$$F\leq\int d\xi\ \mathrm{tr}\left(\frac{E(\xi)\rho_\theta}{\mathrm{tr}(E(\xi)\rho_\theta)}\right)\cdot\mathrm{tr}(E(\xi)L^2\rho_\theta) \tag{8}$$

leading to

$$F_Q=\int d\xi\ \mathrm{tr}(E(\xi)L^2\rho_\theta)=\mathrm{tr}(L^2\rho_\theta)=\langle L^2\rangle. \tag{9}$$

and the second inequality in (3) is saturated. This inequality circumvents the maximization problem by placing an upper bound on F_Q in terms of the expectation value of the square of the symmetric logarithmic derivative operator L. This expectation value can be computed directly from the initial state of the probe and its parameter dependent dynamics, independent of the read-out procedure.

We choose to compare the precision of the parameter estimation for the QDS state using this widely accepted approach of QFI. The interferometric set-up generally consists of four steps. The first is the preparation step where the input state is chosen as an isolated electron state, ρ_{int} in the QDS (here the isolated electron is assumed to be a qubit system). Then, a singlet-qubit phase gate is applied, given by

$$U_\theta:=|g\rangle\langle g|+e^{i\theta}|e\rangle\langle e|. \tag{10}$$

The outcome state is obtained as the output during a uniform process, $\rho_{\mathrm{out}}=U_\theta\rho_{\mathrm{int}}U_\theta^\dagger$. After the phase gate operation, the output mixed state $\rho_{\mathrm{out}}(\phi)$ is finally measured for the estimation of phase uncertainty.

The entangled N-qubit states have been proposed as means to beat the so-called shot-noise limit accuracy in parameter estimation [21,22]. Indeed, if the parameter θ appears in the transformation U_θ, one can measure by subjecting a system in an initial state ρ_{int} to the unitary operator U_θ. The QCR inequality provides a lower limit to the accuracy of estimation in terms of the inverse of the square of the QFI associated with the generator of the unitary transformation and the state of the system. Now, if ρ_{in} is a separable state, the QFI scales as $O(N)$ with the number of particles in the system, N, while it may scale faster for entangled ρ_{int}.

3. Quantum dot system

We consider a quantum dot where the charging energy E_C is the largest energy scale of the problem, which is the experimentally relevant situation. For a dot with completely broken spatial symmetries such as a lateral quantum dot, random matrix theory applies and the single-particle energy levels ε_n are statistically distributed following a Wigner–Dyson ensemble. On the average, the mean level spacing δE is much smaller than E_C. Furthermore, we take into account intradot exchange interactions that favor ferromagnetic configurations if the dot electron number is even, similar to the Hund's rule in atomic physics. The strength of this interaction, $E_S>0$, satisfies $E_S\ll\delta E\ll E_C$. Then, the quantum-dot universal hamiltonian reads [23]

$$H_{\mathrm{dot}}=\sum_{ns}\varepsilon_n d_{ns}^{+}d_{ns}-E_S S_{tot}^2-E_z S^z+E_C(N-N_0) \tag{11}$$

where $N=\sum_{ns}d_{ns}^\dagger d_{ns}$ is the total number of electrons in the dot and N_0 can be adjusted with a nearby gate voltage [24]. In what follows, we assume that the dot is tuned into a Coulomb-blockade valley with an even integer electron number ($N_0=2$). Hence, we label the two active orbital levels with $n=-1$ and $n=+1$. For $E_S\ll\delta E$, the dot total spin,

$$S=\frac{1}{2}\sum_{nss'}d_{ns}^\dagger\sigma_{ss'}d_{ns'}^\dagger \tag{12}$$

is 1/2 for N odd and 0 for N even. However, since $\delta=\varepsilon_{+1}-\varepsilon_{-1}>0$ can be random, a singlet–triplet transition will occur when E_S of the order of δ. Alternatively, δ can be controlled using an externally applied magnetic field acting on the electronic orbital motion.