ELSEVIER

Journal of Materials Processing Technology 175 (2006) 225–230

Journal of Materials Processing Technology

www.elsevier.com/locate/jmatprotec

Actor-initiated snap-through of unsymmetric composites with multiple deformation states

W. Hufenbach*, M. Gude, A. Czulak

Technische Universität Dresden, Institut für Leichtbau und Kunststofftechnik (ILK), D-01062 Dresden, Germany

Abstract

Composites with an unsymmetric lay-up are generally characterized by warpage phenomena due to residual stresses caused in the manufacture process or during operating conditions. For special classes of unsymmetric, fibre-reinforced and textile reinforced composites more than one stable deformation state occurs due to the residual stresses caused by thermal effects, moisture absorption and chemical shrinkage. In the case of the existence of bistable equilibrium states a snap-through from one stable deformation state to the second state is possible, which can be initiated by forces and moments generated by adapted actors. Thus, novel adaptive structure can be developed, which enable a shape adaption only by a short actor impuls.

This paper focuses on the formulation of theoretical fundamentals for the design of those novel adaptive structure. A non-linear calculation method is presented, which serves as a basis for the prediction of warpage and for the determination of mono- and multistable deformation states of unsymmetric composites. Based on various parameter studies, which reflect the possibilities of the design of multistable laminates, first prototypes of adaptive structures with multiple stability states have been developed and successfully tested.

Keywords: Multistable composites; Residual stresses; Snap-through; Adaptive structures; Actor

1. Introduction

During the manufacture of multilayered fibre- and textile-reinforced composites with variable fibre orientations, residual stress states arise due to the directional expansion of the unidirectionally (UD) reinforced single layers or the basic textile layers. Dependent on the laminate lay-up and the textile architecture, these inhomogeneous residual stresses, which are primarily caused by thermal effects, moisture absorption and chemical shrinkage, can lead to large mono- and multistable out-of-plane deformations in the case of unsymmetric laminates (Fig. 1).

Instead of avoiding these laminate's curvatures and the belonging multistable deformation states, they can be advantageously used for technical applications such as novel adaptive structures. In order to adjust the laminate deformations to technical requirements, a dimensioning tool based on a modified stability analysis has been developed and experimentally verified. With the help of the theoretical model, adaptive prototypes of multistable composites with integrated actors have been designed, manufactured and tested.

2. Non-linear deformation theory of multistable composites

For unsymmetric laminates, the hygro-thermally and chemically caused directional deformations of the UD single layers result in large out-of-plane deformations. To take into account the large multistable deformations, which are often many times over the laminate thickness, the linear strain–displacement relations must be extended by non-linear terms (see e.g. [1–4]):

$$\varepsilon_x = \varepsilon_x^0 - z\frac{\partial^2 w}{\partial x^2} = \frac{\partial u^0}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2},$$

$$\varepsilon_y = \varepsilon_y^0 - z\frac{\partial^2 w}{\partial y^2} = \frac{\partial v^0}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 - z\frac{\partial^2 w}{\partial y^2},$$

* Corresponding author. Tel.: +49 351 463 38142; fax: +49 351 463 38143.
E-mail address: ilk@ilk.mw.tu-dresden.de (W. Hufenbach).

0924-0136/$ – see front matter © 2006 Published by Elsevier B.V.
doi:10.1016/j.jmatprotec.2005.04.025

Fig. 1. Multistable deformation states of unsymmetric composites.

$$\varepsilon_{xy} = \varepsilon_{xy}^{0} - z\frac{\partial^2 w}{\partial x \partial y} = \frac{1}{2}\left(\frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\right) - z\frac{\partial^2 w}{\partial x \partial y},$$

where the index 0 refers to the laminate reference plane.

The developed dimensioning tool for unsymmetric composites is based on the principle of minimising the total

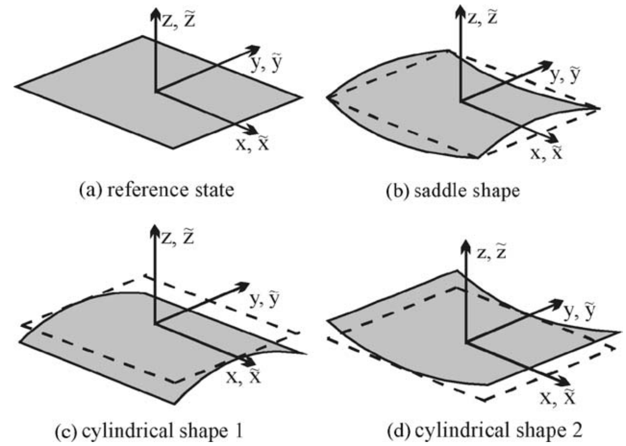

Fig. 2. Basic shapes of square $[0_n/90_m]$ laminates: (a) reference state at elevated curing temperature; (b–d) saddle and cylinder shapes at room temperature.

potential energy, which is given here by

$$\Pi = \int_V \left(\frac{1}{2}\bar{Q}_{ij}\varepsilon_i\varepsilon_j - \eta_{\mathrm{T}i}\varepsilon_i\Delta T - \eta_{\mathrm{M}i}\varepsilon_i\Delta M - \eta_{\mathrm{S}i}\varepsilon_i\right)\mathrm{d}V$$

with $i, j = 1, 2, 6$, where the $\bar{Q}_{ij}$ are the reduced transformed stiffnesses, ΔT and ΔM are the differences in temperature and relative media concentration with respect to the reference state and $\eta_{\mathrm{T}i}$, $\eta_{\mathrm{M}i}$, η_{Si} are related to the elastic constants and to the thermal expansion coefficients α_j ($\eta_{\mathrm{T}i} = \bar{Q}_{ij}\alpha_j$), the swelling coefficients β_{j} ($\eta_{\mathrm{M}i} = \bar{Q}_{ij}\beta_j$) and the shrinkage s_j ($\eta_{\mathrm{S}i} = \bar{Q}_{ij}s_j$), respectively [5].

Based on the total potential energy, the Rayleigh–Ritz method is applied to obtain approximate solutions for the resulting displacement fields. Therefore, general approaches in the form of polynomials are used dependent on the laminate lay-up and textile structure [4].

For a square cross-ply $[0_n/90_m]$ laminate the occurring basic shapes are illustrated in Fig. 2. Starting from the plane shape, which is considered as the reference state (Fig. 1a), the residual stresses lead – dependent on the laminate dimensions and non-mechanical loads – to a saddle shape (b) or to either of the two stable cylindrical shapes (c and d) (see also [5]). For $[0_n/90_m]$ laminates with $n \ll m$ or $n \gg m$, however only one stable cylindrical shape occurs.

The associated approach for the out-of-plane deformation is chosen to

$$w(x, y) = \frac{1}{2}(a_0 x^2 + b_0 y^2),$$

where the unknown coefficients a_0 and b_0 define the laminate curvatures along the x and y axes. Based on adapted displacement approaches for the displacement u and v with further unknown Ritz coefficients the principle of the minimum total potential energy in combination with the Rayleigh–Ritz method leads to a non-linear equation system, which is