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Vibration analysis of corrugated beams: The effects of temperature and corrugation shape

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ABSTRACT

This paper deals with the vibration analysis of corrugated beams under large amplitude displacements and temperature variations. The dynamic response depends on the shape, the period and the amplitude of the corrugations as well as on the temperature variations. These different parameters were taken into account using a homogenization process. We coupled the harmonic balance method and the Galerkin technique to derive a frequency amplitude equation, which includes the corrugation shape and temperature. Finally, the influence of different shapes of corrugation (sinusoidal, triangular and square) on the nonlinear vibration response of a corrugated beam was compared to results obtained for a planar beam.

1. Introduction

Vibration analysis is an important issue in many areas of structural engineering such as automotive, aeronautics and fabrication industries. The amplitude of vibrations becomes large around natural frequency. For thin structures, this is due to geometrical nonlinearities, which provide nonlinear differential equations in space and time. Studying such vibrations is important as it can damage structures through jump phenomena or instability. Moreover, large amplitude vibrations can produce noise.

Different methods have been proposed to account for nonlinear vibrations in particular for planar shape. Many authors have written about the impact of large amplitude vibrations on elastic planar beams or plates by giving analytical solutions such as Pirbadaghi et al. [1]; Azrar et al. [2]; Hoseini et al. [3], Benamar et al. [4], or by using numerical techniques (Shi and Mei [5]).

In the context of energy saving, the minimization of weight is becoming of major importance in the automotive industries. Therefore, corrugated structures have been used for weight reduction and optimal rigidity purposes. Corrugated beams have been investigated mainly under static conditions. In this case, Potier-Ferry et al. [6], Mouftakir [7] and Cartraud and Messager [8] derived equivalent tension–compression and bending stiffness moduli taking corrugation geometry into account. In their work, the homogenization process was carried out for zigzag and sinus beams.

Temperature can also considerably affect a structure's vibration response. The thermal effect on nonlinear vibrations of rectangular plates with clamped edges was introduced by Amabili [9]. Liew et al. [10] studied the impact of the buckling of corrugated plates using a mesh-free Galerkin method.

However, the effects of corrugation shape and temperature on the nonlinear vibration analysis have not been investigated yet. In this paper, we propose an analytical approach for the nonlinear analysis of corrugated beams, which includes both temperature variations, and different corrugation's shapes. The nonlinear behaviors of corrugated beams are investigated around the first resonance frequency.

2. Theoretical framework

In this study, the isotropic 2D beams are considered with uniform thickness and periodic wave corrugations. The beams are uniformly corrugated in the direction of the projected length L. These corrugated beams are designed as planar beams with uniform thickness. The homogenized moduli (tension–compression

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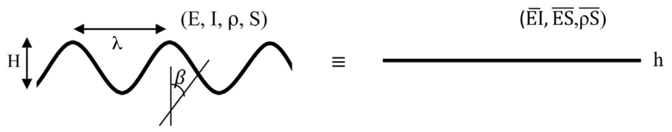


Fig. 1. Shape of the sinusoidal corrugated beam.

and bending stiffness moduli) and the equivalent linear density are respectively denoted ($\overline{EI}$, $\overline{ES}$) and ($\overline{\rho S}$).

Based on the Euler–Bernoulli's beam model, these equivalent constants of the corrugated beam are obtained using the geometrical homogenization technique. In this paper, only the transverse inertia effect is accounted and the longitudinal and rotatory inertia effects are neglected because, only the flexural harmonic forces are considered. The resolution of the nonlinear model equations is achieved using the harmonic balance method and the Galerkin technique. Thus, a nonlinear differential equation is derived providing the response curves of the nonlinear vibrations taking into account both shape and thermal effect.

2.1. Homogenization methods

Using a geometrical homogenization approach, Potier-Ferry et al. [6] defined the equivalent traction compression stiffness modulus and the bending stiffness modulus by taking the geometry into account (Fig. 1). Thus, the uniform flat corrugated beam can be defined by its mid-line of length L. The mid surface of the corrugated beam is a plane curve characterized by an arc curvilinear abscissa δ and a λ-period along x-axis. λ is assumed to be "small" with respect to L.

According to the classical Euler–Bernoulli's beam theory and the macroscopic model, the equivalent traction stiffness and the bending stiffness modulus are given by:

$$\left(\frac{1}{\overline{ES}}\right) = \frac{\cos^2\beta}{ES\cos\beta} + \frac{z^2}{EI\cos\beta} \quad \text{and} \quad \overline{EI} = EI\cos\beta$$

where the average operator is given by: $\langle . \rangle = \frac{1}{\lambda}\int_0^\lambda (.)\, ds$

This formulation can be used for any form of beam corrugation. Thus, three shapes are studied: sinusoidal, triangular and square corrugation. For the different geometrical shapes, the equivalent length (l_d) of a period of corrugation and the corresponding stiffness are given in Table 1.

In order to compare accurately the results from these different corrugated beams, the mass of the samples is assumed to be equal (in return, thickness are different). The equivalent linear density is then given by:

$$\overline{\rho S} = \frac{\rho S l_d}{\lambda} \tag{2}$$

2.2. Motion equations

In this study, the dynamic problem is considered using a 2D semi-analytical approach. The equation of motion for any conservative system with n degrees of freedom q_i $(i = 1, \ldots, n)$ can be expressed using Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial C}{\partial \dot{q}_i}\right) - \frac{\partial C}{\partial q_i} + \frac{\partial U}{\partial q_i} = \frac{\partial W}{\partial q_i} \tag{3}$$

where C is the kinetic energy, U is the elastic strain energy and W is the work of generalized forces.

Considering the linear elasticity, Hooke's law describes the relation between stress and strain, taking into account thermal expansion. For a uniform beam, these formulations are in given in the following in the case of the large deflections. The axial resultant force and the bending are respectively denoted by (N_x) and (M_z)

$$\sigma_x = E(\varepsilon_x - \alpha\Delta T) \tag{4}$$

$$\varepsilon_x = \frac{\partial u}{\partial x} - y\frac{\partial v}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2 \tag{5}$$

$$N_x = \int_S \sigma_x dS \tag{6}$$

$$M_z = \overline{EI}\left(\frac{\partial^2 v}{\partial x^2}\right) \tag{7}$$

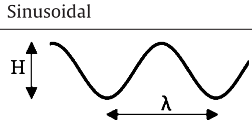
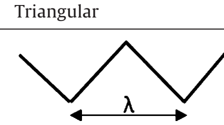
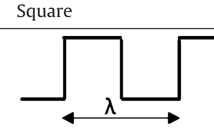
where u is the longitudinal displacement, v is the transversal displacement, α is the coefficient of thermal expansion and ΔT is the thermal variation of the mid surface of the beam. In this study, the geometrical effects are considered using the Von Karman's theory, which assumes moderate rotations and small axial displacements, thus the term $\left(\frac{\partial u}{\partial x}\right)^2$ can be neglected.

$$N_x = \int_S E\left(\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 - \alpha\Delta T\right) dS \tag{8}$$

$$N_x = \int_S \frac{\partial u}{\partial x} EdS + \frac{1}{2}\int_S \left(\frac{\partial v}{\partial x}\right)^2 EdS - \alpha\Delta T\int_S EdS \tag{9}$$

$$N_x = \overline{ES}\frac{\partial u}{\partial x} + \frac{\overline{ES}}{2}\left(\frac{\partial v}{\partial x}\right)^2 - \alpha\Delta T\overline{ES} \tag{10}$$

Table 1
Equivalent length on corrugation one duration and the corresponding stiffness for the geometrical shapes considered.

Shape	Sinusoidal	Triangular	Square
Diagram			
l_d	$\int_0^\lambda \sqrt{1+\left(\frac{\pi H}{\lambda}\cos\left(\frac{2\pi x}{\lambda}\right)\right)^2}dx$	$\sqrt{\lambda^2+4H^2}$	$\lambda+2H$
$\overline{ES}$	$\left(\frac{1}{\lambda ES}\int_0^\lambda \frac{dx}{\sqrt{1+\left(\frac{\pi H}{\lambda}\cos\left(\frac{2\pi x}{\lambda}\right)\right)^2}} + \frac{H^2}{4\lambda EI}\int_0^\lambda \sin^2\left(\frac{2\pi x}{\lambda}\right)\sqrt{1+\left(\frac{\pi H}{\lambda}\cos\left(\frac{2\pi x}{\lambda}\right)\right)^2}dx\right)^{-1}$	$\left(\frac{\lambda}{ESl_d}+\frac{l_dH^2}{12\lambda EI}\right)^{-1}$	$\left(\frac{1}{ES}+\frac{1}{EI}\left(\frac{H^3}{6\lambda}+\frac{H^2}{4}\right)\right)^{-1}$
$\overline{EI}$	$\frac{\lambda EI}{l_d}$		