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Numerical approach for detecting bifurcation points of the compatibility paths of symmetric deployable structures

Yao Chen, Jian Feng*, Zheng Ren

Key Laboratory of Concrete and Prestressed Concrete Structures of Ministry of Education, and National Prestress Engineering Research Center, Southeast University, Nanjing 210096, China

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ABSTRACT

New internal mechanisms of a deployable structure could be generated, when the structure undergoes significant transformations along its compatibility path. Because of such kind of kinematic bifurcation, the structure might not transform into the desired configuration. To design novel deployable structures, it is necessary to detect all possible bifurcation points of the compatibility paths and study the bifurcation behavior. Here, on the basis of the nonlinear prediction–correction algorithm with variable increment size, we will propose an efficient approach to detect all the possible bifurcation points of the compatibility path for a symmetric deployable structure. Null space of the Jacobian matrix is studied iteratively, to follow the complete compatibility path. The variable increment size at each step is determined by evaluating whether the configuration is close to the singular configuration. Numerical examples of several 2D and 3D symmetric deployable structures are presented, to verify the feasibility and computational complexity of the proposed approach. The results show that the proposed method is computationally efficient, and could detect different bifurcation points of the compatibility path. Further, it turns out that all the analyzed symmetric structures experience kinematic bifurcation on certain conditions.

1. Introduction

Deployable structures are a type of novel kinematically indeterminate structures that have internal mechanisms and can be transformable. As these structures have strong vitality, they have been gradually applied in aerospace engineering, mechanical engineering and civil engineering. However, new internal mechanisms of a deployable structure could be generated at bifurcation, when the structure is folding along its compatibility path. Reaching a bifurcation point of the compatibility path, such a structure can transform along either the original path or the secondary paths at bifurcation. Thus, the structure might not transform into the desired configuration. Then, the geometric configuration, mobility and kinematic indeterminacy can change suddenly, which is a complicated and nonlinear behavior. Therefore, it is important and necessary for detecting all possible bifurcation points of the compatibility paths, to design novel deployable structures.

In fact, kinematic bifurcation exists for many finite mechanisms, especially symmetric deployable structures with pin-joints or revolute hinges [1–4]. This phenomenon has attracted great attention. Some numerical approaches have been described and utilized to follow the complete compatibility path of a deployable structure [5–9]. To detect kinematic bifurcation and predict kinematic behavior of deployable pin-jointed structures, Kumar and Pellegrino [10] gave a computational scheme using the singular value decomposition (SVD) of the equilibrium matrix. It turns out that when a deployable structure is getting close to a bifurcation point along the compatibility path, the involved matrices (e.g., the equilibrium matrix and the compatibility matrix) further become numerically ill-conditioned, and the smallest nonzero singular value would successively decrease to zero. Lengyel and You [11] studied the singularities of pin-jointed structures with a single degree of freedom using the elementary catastrophe theory. They found that common cuspoid catastrophe types could exist in the compatibility paths of finite mechanisms with pin-joints. More recently, Yuan et al. [12] proposed an analogous stiffness method to investigate the kinematic bifurcation of finite mechanisms. The bifurcation points along the compatibility paths of finite mechanisms could be detected by solving analogous stiffness equations and compatibility equations simultaneously.

However, most of the aforementioned methods are focused on kinematic analysis of pin-jointed structures. In fact, there are many other types of deployable structures connected by linkages and revolute joints. Based on the SVD of the Jacobian matrix of

* Corresponding author. Tel.: +86 025 83793150; fax: +86 025 83373870.
E-mail address: fengjian@seu.edu.cn (J. Feng).

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constraint equations, Chen and You [13] discovered that kinematic bifurcations could occur along the deployments of a type of symmetric foldable frame. Some researchers have studied the Jacobian matrix method and investigated kinematic characteristics at singular configurations of overconstrained mechanisms [14,15]. Macho et al. [16] introduced a general symmetrical procedure to Jacobian matrix method to trace singularity maps of parallel manipulators.

Here, based on the nonlinear prediction–correction algorithm [10,17], we will present an efficient numerical approach with variable increment size. It can detect all the possible bifurcation points of the compatibility path for a symmetric deployable structure, which is either a pin-jointed structure or an overconstrained mechanism. Null space of the Jacobian matrix is iteratively evaluated to follow the compatibility path. The variable increment size for each step is determined by evaluating whether the configuration is getting close to the singular configuration.

2. Numerical approach for identifying singularity

2.1. Kinematic constraint equations

Deployable structures presented throughout this study are mainly assembled by rigid links via pin-joints or revolute joints. Before constructing the Jacobian matrix for a deployable structure, we describe the kinematic constraint equations for typical constraints. As shown in Fig. 1(a), rigid links are commonly used as connecting elements, and their lengths keep invariant regardless of finite motions [18].

Given a rigid link element connected by two nodes i and j, the nonlinear kinematic constraint equation can be written as:

$$F_k(\boldsymbol{X}_i, \boldsymbol{X}_j) = (\boldsymbol{X}_j - \boldsymbol{X}_i) \cdot (\boldsymbol{X}_j - \boldsymbol{X}_i)^{\mathrm{T}} - L_{ij}^2 = 0 \quad (1)$$

where $F_k()$ is generalized as a function for describing the kinematic constraints, L_{ij} is the length of the rigid link, and $\boldsymbol{X}_i$ and $\boldsymbol{X}_j$ are the row vectors of the nodal coordinates of i and j. For example, for a node i in a 3D Cartesian coordinates system, the vector

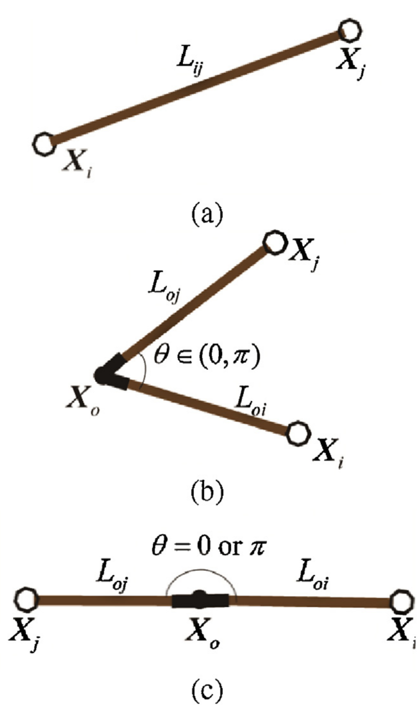

Fig. 1. Typical kinematic constraints: (a) rigid link; (b) constant angle $\theta \in (0, \pi)$; (c) constant angle $\theta = 0$ or π.

$\boldsymbol{X}_i = [x_i, y_i, z_i]$, and x_i, y_i, and z_i are the coordinates of the node in the x, y and z directions, respectively.

Fig. 1(b) and (c) shows the angular constraints, which are generally adopted for scissor-like elements [19] and other overconstrained mechanisms [4,7]. The angular constraint is of the form:

$$F_k(\boldsymbol{X}_o, \boldsymbol{X}_i, \boldsymbol{X}_j) = (\boldsymbol{X}_i - \boldsymbol{X}_o) \cdot (\boldsymbol{X}_j - \boldsymbol{X}_o)^{\mathrm{T}} - L_{oi} L_{oj} \cos \quad \theta = 0 \quad (2)$$

where $\boldsymbol{X}_o$ is the row vector for the intersecting node o, L_{oi} and L_{oj} are the lengths of the two adjacent link segments, and θ is the angle between the two link segments. However, when the two adjacent link segments in Fig. 1(c) are parallel (i.e., $\theta = 0$ or π), the kinematic constraint equation should be rewritten as:

$$F_k(\boldsymbol{X}_o, \boldsymbol{X}_i, \boldsymbol{X}_j) = \left\| (\boldsymbol{X}_i - \boldsymbol{X}_o) \otimes (\boldsymbol{X}_j - \boldsymbol{X}_o) \right\|_2 - L_{oi} L_{oj} \sin \quad \theta = 0 \quad (3)$$

where $\otimes$ defines the tensor product of two vectors, and $\left\| (\boldsymbol{X}_i - \boldsymbol{X}_o) \otimes (\boldsymbol{X}_j - \boldsymbol{X}_o) \right\|_2$ is the 2-norm of $(\boldsymbol{X}_i - \boldsymbol{X}_o) \otimes (\boldsymbol{X}_j - \boldsymbol{X}_o)$.

In addition, it is easy to write the boundary constraint equation for a fixed node i as:

$$F_k(\boldsymbol{X}_i) = \boldsymbol{X}_i - \mathrm{C} = 0 \quad (4)$$

where the row vector $\mathbf{C}$ keeps constant.

Then, the kinematic constraint equations for the whole structure can be assembled in a similar way, where the kth constraint equation is expressed in a compact form as:

$$F_k(\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3, \ldots, \boldsymbol{X}_n) = 0 \quad \text{for}\ k = 1, 2, \ldots, n_c \quad (5)$$

In Eq. (5), n_c is the number of kinematic constraints.

2.2. The Jacobian matrix and the internal mechanism modes

After taking the derivative of the constraint equations, the resulting equation can be written as:

$$\boldsymbol{J} \cdot d\boldsymbol{X} = 0 \quad (6)$$

The kth row of the Jacobian matrix is

$$\boldsymbol{J}_k = \left[\frac{\partial F_k(\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3, \ldots, \boldsymbol{X}_n)}{\partial \boldsymbol{X}_1}, \ldots, \frac{\partial F_k(\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3, \ldots, \boldsymbol{X}_n)}{\partial \boldsymbol{X}_n} \right] \quad \text{for}\ k = 1, 2, \ldots, n_c \quad (7)$$

and $d\boldsymbol{X} = [d\boldsymbol{X}_1, d\boldsymbol{X}_2, \ldots, d\boldsymbol{X}_n]^{\mathrm{T}}$. Note that the Jacobian matrix contains some important information which can be utilized in kinematic analysis [20]. For instance, the null space of the Jacobian matrix gives the independent internal mechanism modes, denoted as a matrix $\boldsymbol{M}$. Then the number of internal mechanism modes for a deployable structure with boundary constraints is:

$$m = d \times n - r \quad (8)$$

where n is the number of generalized nodes in a structure, and r is the rank of the matrix $\boldsymbol{J}$. The dimension of a generalized node for a 2D pin-jointed structure, 3D pin-jointed structure, 2D overconstrained structure or 3D overconstrained structure is $d = 2$, 3, 3 or 6, respectively. In addition, the number of internal mechanisms for a free-standing deployable structure is:

$$m = d \times n - r - T - R \quad (9)$$

where T is the dimension of rigid-body translations, and R is the dimension of rigid-body rotations. Accordingly, $T + R$ modes of rigid-body motions have been excluded from the mechanism mode matrix $\boldsymbol{M}$ [21].

On the other hand, when a deployable structure is expressed in the symmetry-adapted coordinate system, the Jacobian matrix is of the block-diagonalized form [22–24]. Null spaces of the blocks of