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Nonlocal or gradient elasticity macroscopic models: A question of concentrated or distributed microstructure

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ABSTRACT

One dimensional discrete systems, such as axial lattices, may be investigated by using some enriched continuum models. In this paper, strain gradient models (also called gradient elasticity) and stress gradient models (also called nonlocal elasticity) are both shown to be supported by some microstructured physical configurations. Starting from the difference equations associated with each discrete system, a continualization approach is applied to the governing difference equations. Alternatively, one may use energy considerations to derive these higher-order continua. Stress gradient models are built from concentrated microstructure (with direct neighboring interaction) whereas strain gradient models are associated to some distributed microstructure (also with direct neighboring interaction). Each model leads to opposite effect, namely the softening effect of the small scale terms for the stress gradient model (built from concentrated microstructure), and the stiffening effect of the small scale terms for the strain gradient model (built from distributed microstructure) with respect to the asymptotic local model. We also discuss the link between lattice equations, finite difference formulation or finite element formulation of the continuous local problem. The paper concludes that the local neighboring interaction at the discrete scale may transmit some higher-order effects at the macroscopic scale. Hence, the higher-order nature of the macroscopic constitutive laws may not necessarily be seen as the consequence of nonlocal interaction at the lattice scale.

1. Introduction

Discrete repetitive systems or lattice systems may be analyzed using continuum models for some deeper investigations of their behaviors. There are less theoretical results available for discrete systems characterized by difference equations when compared to the more mature theory of differential or partial differential equations. It is of primary interest for engineering purposes to be able to connect lattice mechanics with continuum mechanics. Lagrange [1] was apparently the first one who showed the link between one-dimensional lattices (string lattice or axial lattice) with the associated continua, which is asymptotically obtained for infinite number of elements. Piola during the *XIX*th century built peridynamics (or relative displacement-based) nonlocal models and higher-order gradient continua from discrete microscale interactions [2,3]. It is not the scope of this paper to report the very rich literature dedicated to the link between lattice and continuum models, especially for truss-type or beam-type lattices (see for instance [4–8]). We will mainly focus our analysis on one-dimensional stress gradient models and strain-gradient models, where both are reputed to be able to capture small scale effect induced by the microstructure at a subscale.

There is a debate in the literature between these two families of enriched continua, namely the stress gradient model which is also called the nonlocal model [9] and the strain gradient model which is also called gradient elasticity model [10,11]. These enriched continuum models lead to opposite results, namely the softening effect of the small length scale for the stress gradient model, and the stiffening effect of the small length scale for the gradient elasticity model [12,13]. A further elucidation of this apparently paradoxical situation is the explicit aim of this investigation.

One possible way to differentiate each model is to derive some micromechanics arguments supporting each class of model. It has been already shown by Eringen [9] that axial lattices with concentrated microstructure and direct neighboring interaction,

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may be efficiently fitted by the stress gradient model (or the nonlocal model). Eringen [9] calibrated the length scale parameter of the stress gradient model by comparing the wave dispersive properties of the nonlocal with the lattice one, also referred to as the Born–Kármán lattice model [14]. It is also possible to calibrate this length scale parameter from the lower frequency spectrum, as shown by Challamel et al. [15] for the axial lattice system. One can state that nonlocal models may be introduced from lattice discrete arguments that are only associated with direct neighboring interactions (see also [16] for bending lattice systems). It is also possible to build the nonlocal kernel from generalized lattice interactions (see [17] for strain-based integral model or [18] for peridynamics-type nonlocal model).

There are, however, some other arguments for justifying gradient elasticity models. Mindlin [19] showed that gradient elasticity models may be asymptotically derived from lattice systems with direct and indirect neighboring interactions (including two and three-neighbor interactions), as already considered by Gazis and Wallis [20]. This idea was used again by Polyzos and Fotiadis [21] or more recently by Polyzos et al. [22] for deriving gradient elasticity models with direct and indirect interaction models. Carcaterra et al. [23] derived higher-order gradient elasticity models from generalized lattice interactions. Polyzos and Fotiadis [21,22], following the pioneer work of Mindlin [10] also introduced some distributed microstructure or the concept of averaging over a representative volume element [24]. At the same time it appears that the foundation of gradient elasticity model from direct neighboring interaction is still an open and controversial topic that will be discussed herein.

In this paper, we consider the axial lattice with concentrated microstructure as a paradigmatic one dimensional system. The equations are formally identical to those for the torsional lattice or for the string lattice (see [15] for the complete presentation of these three structural problems). Lattice equations with concentrated microstructure are shown to be equivalent to the finite difference formulation of the asymptotic local continuous problem, whereas the lattice equations with distributed microstructure are shown to be equivalent to a finite element formulation of the asymptotic continuous problem with linear interpolation field for the displacement.

We show in this study that the stress gradient nonlocal model is a relevant theory for capturing the scale effect of the lattice model with a concentrated microstructure. The vibration of the nonlocal bar has been already explored by Aydogdu [25] and also Challamel et al. [26] and these nonlocal results efficiently captured the lattice model in this case. On the other hand, we also show that gradient elasticity results applied to the bar (see for instance [11] or [27]) are relevant for the lattice model with distributed mass properties. Concentrated or distributed mass properties lead to different macroscopic properties, namely the softening and the stiffening scale effect with respect to the local continuous model. These lower bound and upper bound status of the nonlocal and the gradient elasticity model are highlighted in the context of finite difference and finite element method properties, as already investigated for instance by Polya [28] for the membrane problem.

2. Axial lattice with concentrated microstructure

Consider an axial lattice composed of $n+1$ concentrated masses connected by n linearly elastic springs (see Fig. 1). The axial lattice (Born–Kármán lattice model with direct neighboring interactions) is composed of n repetitive cells of length denoted by $a=L/n$, where L is the total length of the axial chain. This model can be also labeled as a Lagrange lattice since Lagrange investigated the discrete string as well as the axial lattice and outlined the formal equivalence between both systems [1]. The cell length a may be related to the microstructure of this lattice model, in connection with interatomic distance or to some other microstructured length. Fixed–fixed (Fig. 1a) and Fixed-free (Fig. 1b) boundary conditions are investigated.

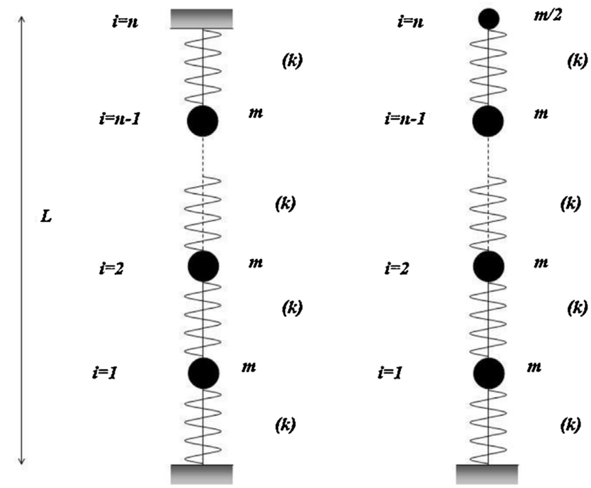

Fig. 1. Axial lattice in vibrations with concentrated microstructure. (a) Fixed–fixed boundary conditions; (b) Fixed-free boundary conditions.

The elastic potential W of this discrete chain is given by

$$W=\sum_{i=1}^{n-1}\frac{EA}{4}\left[\left(\frac{u_{i+1}-u_i}{a}\right)^2+\left(\frac{u_i-u_{i-1}}{a}\right)^2\right]\times a+\frac{EA}{4}\left(\frac{u_1-u_0}{a}\right)^2\times a+\frac{EA}{4}\left(\frac{u_n-u_{n-1}}{a}\right)^2\times a \quad (1)$$

where the stiffness of each spring k is calibrated such that $k=EA/a$. Of course, it is formally possible to include in the following some analogous mechanical problems such as the string or the torsional problem, with relevant adaptation of notations. It is possible to express the elastic potential in the following equivalent form:

$$W=\sum_{i=0}^{n-1}\frac{EA}{2}\left[\left(\frac{u_{i+1}-u_i}{a}\right)^2\right]\times a \quad (2)$$

Now, considering a lattice system with concentrated mass properties (classical lattice system, as considered by Born–Kármán), the kinetic energy of the discrete system can be written as:

$$T=\sum_{i=1}^{n-1}\frac{1}{2}\rho A\dot{u}_i^2\times a+\frac{1}{4}\rho A\dot{u}_0^2\times a+\frac{1}{4}\rho A\dot{u}_n^2\times a \quad (3)$$

where each mass m is chosen to be equal to $m=\rho A\times a$, except at the boundary nodes where the mass is halved (see also Fig. 1). By applying Hamilton's principle to the lattice system (whose potential energy is given by Eq. (1) and kinetic energy is given by Eq. (2)), one obtains the lattice equations:

$$EA\frac{u_{i+1}-2u_i+u_{i-1}}{a^2}-\rho A\ddot{u}_i=0 \quad (4)$$

This mathematical problem is similar to the vibration problem of the discrete string already investigated by [1,45] (see the discussion in [15]). It appears noteworthy that [29] solved a similar mathematical problem for the dynamics behavior of an n-storey planar shear-type frame. It is worth noting that these difference equations associated with the lattice problem are the finite