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Dynamic responses of a hinged-hinged Timoshenko beam with or without a damage subject to blast loading

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ABSTRACT

The present paper addresses the subject of dynamics and damage detection in Timoshenko beam subjected to blast excitation. The general equation governing the dynamics states of both healthy and damaged hinged-hinged Timoshenko beam is derived. A damage function evolving the location, the sizes, the geometry of the damaged area is proposed and the response to blast loading is investigated. The exploration and the comparison of the dynamics of the damaged and healthy states allow to foresee the degree of the structural damage.

1. Introduction

Early structural damage detection is desirable to prevent structural failure and loss of human life. Non-destructive testing (NDT) methods can complement visual inspections as an objective tool leading to quantifiable results. Current NDT techniques such as ultrasound, X-ray, staining, magnetic particle, and acoustic emission are often limited to observation in a limited area and rely on a presumption of the likely area of damage. Structural damage detection using non-destructive vibration test data has received considerable attention since the last decade [1–4,13]. The occurrence of damage will change the static and dynamic characteristics such as vibration response, natural frequency, mode shape and modal damping, which, in turn, can be used to detect, and quantify damage in many situations of engineering practice, such as mechanical, aerospace, and civil engineering.

In the same way, a wide range of mechanical structures from urban infrastructures and buildings to industrial facilities and protective structures may be subjected to blast loadings due to civilian accidents or from detonation of explosives. In recent decades, due to the demand for higher safety against accidental explosions besides elevated terrorism level, the blast-resistant design of structures has become a necessity to understand. In addition to predict the nonlinear dynamic behavior of blast-loaded on structures and develop appropriate design methods, it is essential to combine theoretical analyzes and the results of well-organized experiments. Moreover due to various geometries of these designs, a suitable mathematical modelling should consider the effects of shear deformation and rotary inertia. Thus, the Timoshenko model for beam theory is required and in that regard Han et al. [5] investigated the full analysis of four models for the transversely uniform beam and found that this model give reasonable results for less complexity [6–8]. In the present research, only the effect of air blast overpressures on the dynamic response of structure is studied [9–11]. The influence of ground shock waves or fragment impacts is not considered. The modelling of explosion in air is simulated using pressure-time variation defined by a specific set of and applied to the structures [15–17]. To carry out our investigations we organized this paper as follows. The general mathematical formalism of hinged-hinged Timoshenko beam without and with damage subjected to blast loading is presented in Section 2, follow by the dynamics response of the system using direct numerical simulation of the base equations along with the modal approach using the Galerkin method. Section 3 gives an evaluation shear strength and the relation between the dynamics prediction and the detection of the damaged. Section 4 summarizes the findings.

2. General mathematical formalism

2.1. Mathematical model for the dynamics of hinged-hinged Timoshenko beam under blast loading

Consider a straight Timoshenko beam of length L, a cross-section A, a volumetric mass density ρ, a damping coefficient γ, a shear

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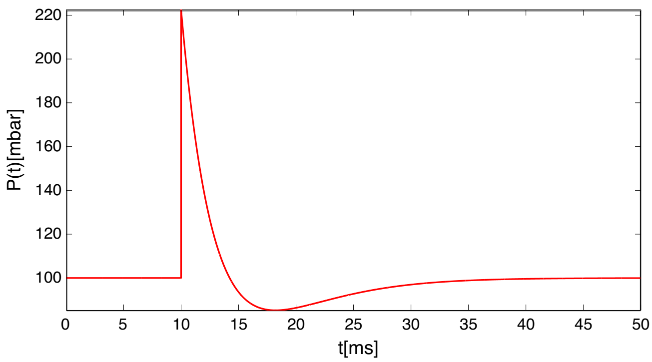

Fig. 1. Ideal blast wave resulting from explosion in air.

factor k, a moment of inertia I, modulus of elasticity and shear E and G. The dynamic equation of motion for the beams in a healthy state is defined as follows [8,18,19]:

$$\rho A\frac{\partial^2 w}{\partial t^2} = kA\frac{\partial}{\partial x}\left[G(x)\left(\frac{\partial w}{\partial x}-\psi\right)\right] - \gamma\rho A\frac{\partial w}{\partial t} - b_0 P(t) \tag{1a}$$

$$\rho I\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial}{\partial x}\left(E(x)I\frac{\partial \psi}{\partial x}\right) + kGA\left(\frac{\partial w}{\partial x}-\psi\right) - \gamma\rho I\frac{\partial \psi}{\partial t} \tag{1b}$$

where $w(x, t)$ and $\psi(x, t)$ are the transverse deflection and the rotation of a beam, and b_0 the width of beam. Eqs. (1a) and (1b) the transverse equation of motion and the second and the rotational equation of motion respectively. For the hinged-hinged Timoshenko beam the following boundary condition are added to the system of equations [18].

$$w(0, t) = w(L, t) = 0 \quad \text{and} \quad \frac{\partial\psi(0, t)}{\partial x} = \frac{\partial\psi(L, t)}{\partial x} = 0$$

$P(t)$ is the air blast loading, $b_0P(t)$ the density of blast loading. It is clear that this equation governs plane flexural vibration along one of the principal direction of the section (then $P(t)$ is a force normal to one principal axis of the section: this is of particular interest for concrete structure that have rectangular section). The principal effects of explosion phenomena on a structure are blast overpressures, fragments impacts and ground shock waves. But this paper addresses only the effect of air blast overpressures on the dynamic response of structures. Usually, the profile of blast shock at a given distance from the detonation point can be represented by an ideal pressure-time curve as shown in Fig. 1, as the blast wave arrives, an almost instant increase from the ambient pressure to the peak overpressure happens. After arrival of the shock front, the overpressure decays down to the ambient pressure exponentially. The first part of wave is termed the positive phase. Then, the overpressure decreases until the negative peak, after which the ambient pressure is obtained once more. The second part of the blast wave is termed the negative phase. The positive phase is usually more important in structural design than the negative phase [20]. According to [11], The mathematical formulation of the blast loading is derived from the modified Friedlander equation and given by [12]

$$P(t) = \begin{cases} P_0, & t < t_a \\ P_0 + P_p\left(1-\dfrac{t-t_a}{t_p}\right)\exp\left(-b\dfrac{t-t_a}{t_p}\right), & t \geq t_a \end{cases} \tag{2}$$

with

$$P_p \quad (Pa) = P_0 \frac{808\left[1+\left(\frac{Z}{4.5}\right)^2\right]}{\sqrt{\left[1+\left(\frac{Z}{0.048}\right)^2\right]}\times\sqrt{\left[1+\left(\frac{Z}{0.32}\right)^2\right]}\times\sqrt{\left[1+\left(\frac{Z}{1.35}\right)^2\right]}}$$

$$t_p \quad (ms) = W^{1/3}\frac{980\left[1+\left(\frac{Z}{0.54}\right)^{10}\right]}{\left[1+\left(\frac{Z}{0.02}\right)^3\right]\times\left[1+\left(\frac{Z}{0.74}\right)^6\right]\times\sqrt{\left[1+\left(\frac{Z}{6.9}\right)^2\right]}}$$

$$b = 1.5Z^{-0.38}$$

where P_0 is the ambient air pressure, P_p is the peak pressure, t_a is the time of arrival at peak pressure, t_p is the duration of blast. W is the charge weight in kg, and Z is the scaled distance and expressed as, $Z = \frac{R}{W^{1/3}}\left(\mathrm{m\,kg}^{-1/3}\right)$. It is also view that the characteristics of air blast wave are functions of the distance to the center of the charge R and the time t.

2.2. Mathematical model of damaged Timoshenko beam

For most practical vibration monitoring problems, it might be difficult to assign a definitive representation for the stiffness of damaged area because the location, sizes, and geometry of the damage are unknown in advance. Besides, due to this complexity, a realistic damage pattern is not always guaranteed if the stiffness along the beam can not vary uniformly. We are looking for a function that can describe a damage pattern by only a few representative parameters. This function should have the flexibility to capture damaged occurring in small and large zones. Assuming that the reduction in the stiffness can be simulated by a reduction of the Young and shear's modulus, this function can be represented by:

$$E_d(x) = E(1 - d(x)) \tag{3a}$$

$$G_d(x) = G(1 - d(x)) \tag{3b}$$

$$d(x) = D_0 sech[\alpha(x - L/2)] \tag{3c}$$

where $E_d(x)$ and $G_d(x)$ are the effective Young and shear's modulus in the damaged state, and $d(x)$ is the damage distribution function which may characterize the state of damage. The case $d(x) = 0$ indicates the healthy state, while $d(x) = 1$ indicates the complete rupture of material due to damage. D_0 and α are the damage parameters. The parameter α characterizes the length of the damaged zone. If α becomes great, a very local damage at the middle of the beam is obtained, whereas if α is small, the beam is damaged over its whole length. D_0 characterizes the magnitude of damage. It varies between 0 and 1. If D_0 varies between 0 and 1 and it characterizes the magnitude of damages (see Fig. 2).

Considering the case of damaged state ($d(x) \neq 0$), the dimensionless form of Eq. (1) is given by

$$\frac{\partial^2 Y}{\partial\tau^2} = (1 - d(X))\left(\frac{\partial^2 Y}{\partial X^2} - \frac{\partial\Psi}{\partial X}\right) - d'(X)\left(\frac{\partial Y}{\partial X} - \Psi\right) - \lambda\frac{\partial Y}{\partial\tau} - k_3P(\tau) \tag{4a}$$

$$\frac{\partial^2 \Psi}{\partial\tau^2} = k_1(1 - d(X))\frac{\partial^2 \Psi}{\partial x^2} - k_1d'(X)\frac{\partial\Psi}{\partial X} + k_2(1 - d(X))\left(\frac{\partial Y}{\partial X} - \Psi\right) - \lambda\frac{\partial\Psi}{\partial\tau} \tag{4b}$$