Mechanics Research Communications 71 (2016) 44–47

Contents lists available at ScienceDirect

Mechanics Research Communications

journal homepage: www.elsevier.com/locate/mechrescom

A Yoffe crack/cohesive zone model for a steady state moving crack

Z.-H. Jin[a,*], C.T. Sun[b]

[a] Department of Mechanical Engineering, University of Maine, Orono, ME 04469, USA
[b] School of Aeronautics and Astronautics, Purdue University, West Lafayyette, IN 47907, USA

ARTICLE INFO

Article history:
Received 16 May 2015
Received in revised form 4 October 2015
Accepted 6 October 2015
Available online 22 October 2015

Keywords:
Crack propagation
Yoffe crack
Dugdale model
Cohesive zone model
Crack opening

ABSTRACT

A Yoffe crack/cohesive zone model is presented to investigate steady state crack propagation in an infinite medium. In the current model, the rear crack tip closes and a cohesive zone exists only at the front crack tip during crack propagation, which eliminates the dilemma in the standard Yoffe crack/Dugdale zone model that the crack length remains constant but the rear crack tip opens up. The cohesive zone length and the crack tip opening displacement (CTOD, the opening at the physical crack tip) are determined in closed forms using a Fourier transform/integral equation approach. It is found that the cohesive zone length and CTOD differ significantly from those in the standard Yoffe crack/Dugdale model in which two cohesive zones exist at both crack tips during crack propagation.

1. Introduction

Cohesive zone models have widely been used to simulate crack extension in materials and structures (see, for example, [1–8]). In a cohesive zone model, a cohesive zone consisting of upper and lower cohesive surfaces is assumed to exist ahead of a crack tip. The cohesive zone behavior is governed by a cohesive law in which the cohesive traction is related to the separation displacement between the cohesive surfaces. Crack extension takes place when the separation at the tail of the cohesive zone reaches a critical value. Clearly, the cohesive zone modeling approach does not involve crack tip stress singularities in classical fracture mechanics, and material failure is controlled by quantities such as displacements and stresses, which are consistent with the usual strength of materials failure theories.

Cohesive zone models have been employed to investigate crack propagation under dynamic loading conditions. For example, Xu and Needleman [9] simulated dynamic crack growth in brittle solids. Camacho and Ortiz [10] investigated impact damage in brittle materials. Falk et al. [11] examined the effect of cohesive law on crack branching in elastic bodies. Zhang and Paulino [12] studied dynamic fracture in functionally graded materials. Elmukashfi and Kroon [13] simulated crack propagation in rubber. Bizzarri [14] computed energy flux at a 3D rupture front propagating with variable speed. Valoroso et al. [15] presented a cohesive zone model with rate-sensitivity for dynamic crack propagation. An important issue in cohesive zone modeling of dynamic crack propagation is verification of the numerical models. Generally speaking, a closed-form analytical solution provides a simple benchmark for verification purposes. However, closed-form solutions are generally not available for dynamic crack propagation problems involving a cohesive zone.

Dugdale model [16] is probably the most simple cohesive zone model for which a closed form solution may be obtained. In the Dugdale model, the cohesive traction is a constant and crack propagation is determined by the crack opening displacement at the physical crack tip. The length of the Dugdale cohesive zone is determined by the condition that the stress singularity at the tip of the Dugdale zone is eliminated. Rice and Simons [17] proposed a Dugdale model for investigating dynamic propagation of a semi-infinite crack under Mode II loading condition. Fan [18] presented a Dugdale model for a Yoffe crack [19]. In the model of Fan [18], two Dugdale zones of equal length exist at the crack tips during crack propagation, an assumption consistent with the Yoffe crack model in which crack length remains unchanged during propagation. It is well known that the Yoffe crack model is not realistic because the rear crack tip would have to close to maintain a constant crack length. Nevertheless, the model [18] provides a closed form solution for dynamic crack propagation using a cohesive zone modeling approach. Most recently, Wu and Ru [20] presented a modified Dugdale model for the Yoffe crack in which the cohesive traction is an arbitrary symmetric function and is determined to satisfy a

* Corresponding author. Tel.: +1 207 581 2135; fax: +1 207 581 2379.
E-mail address: zhihe.jin@maine.edu (Z.-H. Jin).

http://dx.doi.org/10.1016/j.mechrescom.2015.10.002

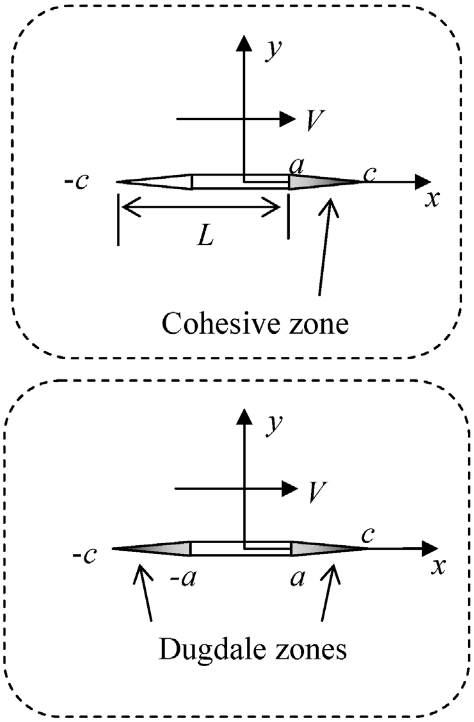

Fig. 1. (a) A moving crack and crack front cohesive zone. (b) The traditional Yoffe crack and Dugdale zones.

yielding criterion along the cohesive zone. Their model, however, still assumed two cohesive zones at both rear and front tips of the moving crack.

The present work considers a Yoffe crack/cohesive zone model in which the rear crack tip closes during crack propagation so that the crack length remains fixed. A Dugdale cohesive zone exists only at the front crack tip and propagates with the crack, see Fig. 1a. The cohesive zone length and the opening at the physical crack tip (tail of the cohesive zone) are determined using a Fourier transform/integral equation approach and are compared with the traditional Yoffe crack model with two Dugdale zones at both tips of the moving crack (Fig. 1b).

2. A Yoffe crack/cohesive zone model

Consider a crack of length L propagating at a constant speed V in an elastic medium as shown in Fig. 1a. The rear crack tip remains closed during propagation and a Dugdale cohesive zone develops at the front crack tip with a length of R. It is assumed that the surfaces of the crack and cohesive zone are subjected to a pressure of linear distribution given by

$$p_0 + p_1 x, \quad -c \le x \le c \tag{1}$$

where p_0 and p_1 are constants, $2c$ is the total length of the crack and the cohesive zone, and x is the moving coordinate with the origin at the center of the crack/cohesive zone. Hence, the crack length and cohesive zone length satisfy the relation

$$R + L = 2c \tag{2}$$

Moreover, the cohesive zone surface is further subjected to the constant cohesive traction as follows

$$\sigma_{cohesive} = \sigma_0, \quad a \le x \le c \tag{3}$$

where $a = c - R$. The boundary conditions of the moving crack/cohesive zone problem are thus given by

$$\sigma_{yy} = -(p_0 + p_1 x) + H(x-a)\sigma_0, \quad |x| \le c, \quad y = 0 \tag{4a}$$

$$u_y = 0, \quad |x| > c, \quad y = 0 \tag{4b}$$

$$\tau_{xy} = 0, \quad |x| < \infty, \quad y = 0 \tag{4c}$$

$$\sigma_{xx}, \sigma_{yy}, \tau_{xy} \to \infty, \quad \sqrt{x^2 + y^2} \to \infty \tag{4d}$$

where $H(\cdot)$ is the Heaviside step function. Finally, the stress intensity factors at the cohesive zone tip $x = c$ and the rear crack tip $x = -c$ must be zero so that no stress singularity exists at the cohesive zone tip and the rear crack tip is closed during crack propagation, i.e.,

$$\begin{aligned} K_I(c) &= 0, \\ K_I(-c) &= 0 \end{aligned} \tag{5}$$

The basic equations of the elastodynamics in the moving coordinate system (x, y) are [6]

$$\begin{aligned} \alpha_1^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} &= 0, \\ \alpha_2^2 \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} &= 0 \end{aligned} \tag{6}$$

The displacements are expressed in terms of the two potentials φ and ψ as follows

$$\begin{aligned} u_x &= \frac{\partial \varphi}{\partial x} + \frac{\partial \psi}{\partial y}, \\ u_y &= \frac{\partial \varphi}{\partial y} - \frac{\partial \psi}{\partial x} \end{aligned} \tag{7}$$

and the stresses are related to the displacements by Hooke's law

$$\begin{aligned} \sigma_{xx} &= \lambda \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + 2G \frac{\partial u_x}{\partial x}, \\ \sigma_{yy} &= \lambda \left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \right) + 2G \frac{\partial u_y}{\partial y}, \\ \sigma_{xx} &= G \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \end{aligned} \tag{8}$$

In Eqs. (6)–(8), λ and G are Lame constants, and α_1 and α_2 are given by

$$\alpha_1 = \sqrt{1 - \left(\frac{V}{c_1}\right)^2}, \quad \alpha_2 = \sqrt{1 - \left(\frac{V}{c_2}\right)^2} \tag{9}$$

where c_1 and c_2 are the wave speeds given by

$$c_1 = \sqrt{\frac{(\lambda + 2G)}{\rho}}, \quad c_2 = \sqrt{\frac{G}{\rho}} \tag{10}$$

in which ρ is the density. Here we assume plane strain conditions prevail. For plane stress, the Lame constant λ is replaced by $\lambda^* = 2\lambda G/(\lambda + 2G)$.

3. Integral equation solution

We use a Fourier transform/integral equation approach to solve the moving crack/cohesive zone problem. The final singular integral equation has the following form

$$\int_{-c}^{-c} \frac{f(x')}{x' - x} dx' = -\frac{\pi F_D(V)}{G} [p_0 + p_1 x - \sigma_0 H(x-a)], \quad -c \le x \le c \tag{11}$$