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Explicit computational method of dynamic response for non-viscously damped structure systems

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ABSTRACT

Non-viscous damping models in which the damping forces depend on the past history of velocities via convolution integrals over some kernel functions have been raised in many engineering fields. This paper describes an explicit computational method of dynamic response for the non-viscously damped structure systems. The explicit formula is derived using the differential property of convolution and the central difference formula of acceleration. The explicit computational procedure of dynamic response is given in detail. Finally, the dynamic responses of MDOF structure system with double exponential model dampers and SDOF structure system with Gaussian model damper are computed using the proposed explicit method. The accuracy and efficiency are discussed by comparison with other two developed methods.

1. Introduction

The non-viscous damping has risen in many engineering fields. For example, noise control in automobiles and airplanes [1], visco-hyperelastic model for filled rubbers [2], passive vibration control of buildings installed with viscoelastic dampers [3], ship dynamics [4]. Adhikari [5] has summarized what type of structure systems may have non-viscous damping. Many non-viscous damping models have been proposed by the researchers in the recent decades to deal with the non-viscous damping. For example, Biot [6,7] damping model, Buhariwala [8] damping model, Bagley and Torvik [9,10] damping models, Golla–Hughes–McTavish [11,12] damping model, anelastic displacement field [13,14] damping model, Gaussian damping model and exponential damping models [15,16]. The non-viscous damping models, in which the damping forces depend on the past history of velocities via convolution integrals over some kernel functions, has been pointed out to be the most general damping model within the linear range by Woodhouse [17]. Therefore, the governing equations of the non-viscously damped structure systems often include convolution integrals.

Direct time-integration methods including the implicit methods [18–20] and explicit methods [21–24] are successful to evaluate the dynamic response of the viscous damping structure systems subjected to complicated dynamic loadings, for example earthquake ground motions [25–28]. As for the non-viscously damped structure systems, Aprile and Benedetti [29] developed a direct integration method for the structures with viscoelastic dampers. Patlashenko et al. [30] developed the time-stepping schemes for the linear Volterra-type systems of integro-differential equations which may arise in the semi-discrete finite element model of dynamic viscoelasticity. Palmeri et al. [31] developed the state space formulas for the linear viscoelastic dynamic systems with memory and proposed the approximated frequency response matrix for the state space formulas. Wagner and Adhikari [32] extended the traditional state-space approach from the viscously damped systems to nonviscously damped systems. Adhikari and Wagner [33] developed a direct time-domain integration method for exponentially damped linear systems based on an extended state space representation of the equations of motion [32]. The method is based on the linear approximations of displacement and velocity. Liu [34] extended Newmark-β method from viscously damped structure systems to non-viscously damped structure systems.

The purpose of this paper is to develop an explicit computational method of the dynamic response for the non-viscously damped structure systems. First, the explicit dynamic response formula is derived using the differential property of convolution and the central difference formula of acceleration. The central difference formula of velocity can be avoided if the damping kernel functions are derivable. Second, the explicit computational procedure of dynamic response is given in detail. Finally, the dynamic responses of MDOF structure system with double exponential model dampers and SDOF structure system with Gaussian model damper are computed using the proposed explicit method in this

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paper. The accuracy, efficiency, merits and limitations are discussed by comparison with other two developed methods.

2. Explicit dynamic response analysis of non-viscously damped structure systems

The governing equations of the non-viscously damped structure systems can be expressed as

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\int_0^t \boldsymbol{G}(t-\tau)\dot{\boldsymbol{x}}(\tau)\,\mathrm{d}\tau+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{f}(t) \tag{1}$$

with initial conditions,

$$\boldsymbol{x}(0)=\boldsymbol{x}_{(0)} \quad \text{and} \quad \dot{\boldsymbol{x}}(0)=\dot{\boldsymbol{x}}_{(0)} \tag{2}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{K}$ is the stiffness matrix, $\boldsymbol{G}(t)$ is a symmetric matrix of the damping kernel functions of the non-viscously damped systems. $\boldsymbol{x}(t)$, $\dot{\boldsymbol{x}}(t)$ and $\ddot{\boldsymbol{x}}(t)$ are the displacement, velocity and acceleration vectors, respectively. $\boldsymbol{f}(t)$ is the loading vector. The convolution integral term is the damping force vector of the non-viscously damped structure systems.

We rewrite Eq. (1) as

$$\boldsymbol{M}\ddot{x}(t)+\boldsymbol{G}(t)*\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=f(t) \tag{3}$$

The damping kernel functions of many non-viscous damping models, i.e., Golla–Hughes–McTavish (GHM) model, anelastic displacement field (ADF) model, Gaussian damping model and exponential damping model, etc., are derivable. The first derivatives of these damping kernel functions with respect to the time variable are also smooth and continuous. Therefore, according to the differential property of convolution, Eq. (3) can also be written as

$$\boldsymbol{M}\ddot{x}(t)+\dot{\boldsymbol{G}}(t)*\boldsymbol{x}(t)+\boldsymbol{K}\boldsymbol{x}(t)=f(t) \tag{4}$$

The central difference formulas of the velocity and acceleration are, respectively,

$$\dot{\boldsymbol{x}}(t)=\frac{1}{2\Delta t}[\boldsymbol{x}(t+\Delta t)-\boldsymbol{x}(t-\Delta t)] \tag{5}$$

$$\ddot{\boldsymbol{x}}(t)=\frac{1}{\Delta t^2}[\boldsymbol{x}(t+\Delta t)-2\boldsymbol{x}(t)+\boldsymbol{x}(t-\Delta t)] \tag{6}$$

where Δt is the sampling time interval.

Compared to Eq. (3), Eq. (4) can avoid using the difference formula of the velocity. Therefore, the convolution term could be simplified and the computational efficiency could be improved. By substituting Eq. (6) into Eq. (4), we have

$$\boldsymbol{M}\boldsymbol{x}(t+\Delta t)=[\boldsymbol{f}(t)-\boldsymbol{K}\boldsymbol{x}(t)-\dot{\boldsymbol{G}}(t)*\boldsymbol{x}(t)]\Delta t^2+\boldsymbol{M}[2\boldsymbol{x}(t)-\boldsymbol{x}(t-\Delta t)] \tag{7}$$

Let the time station $t=i\Delta t\ (i=1, 2, \ldots, n)$, Eq. (7) is rewritten as

$$\begin{aligned}\boldsymbol{M}\boldsymbol{x}_{[(i+1)\Delta t]} &= [\boldsymbol{f}_{(i\Delta t)}-\boldsymbol{K}\boldsymbol{x}_{(i\Delta t)}-\dot{\boldsymbol{G}}_{(i\Delta t)}*\boldsymbol{x}_{(i\Delta t)}]\Delta t\\ &\quad +\boldsymbol{M}\{2\boldsymbol{x}_{(i\Delta t)}-\boldsymbol{x}_{[(i-1)\Delta t]}\}\end{aligned} \tag{8}$$

The rectangular rule of numerical integration is used to calculate the convolution term, Eq. (8) becomes

$$\begin{aligned}\boldsymbol{M}\boldsymbol{x}_{[(i+1)\Delta t]} &= \left\{\boldsymbol{f}_{(i\Delta t)}-\boldsymbol{K}\boldsymbol{x}_{(i\Delta t)}-\sum_{k=0}^{i}\dot{\boldsymbol{G}}_{(k\Delta t)}\boldsymbol{x}_{[(i-k)\Delta t]}\right\}\Delta t^2\\ &\quad +\boldsymbol{M}\{2\boldsymbol{x}_{(i\Delta t)}-\boldsymbol{x}_{[(i-1)\Delta t]}\}\end{aligned} \tag{9}$$

The mass matrix $\boldsymbol{M}$ of the structure system is the diagonal matrix if the lumped mass is employed to model the structure. Therefore, Eq. (9) is the explicit formula for the displacement response. Obviously, Eq. (9) is completely uncoupled and no matrix inversion is required. However, Eq. (9) indicates that $\boldsymbol{x}_{[(i-1)\Delta t]}$ and $\boldsymbol{x}_{(i\Delta t)}$ are needed to be known to calculate $\boldsymbol{x}_{[(i+1)\Delta t]}$.

In order to compute $\boldsymbol{x}_{(\Delta t)}$, we need to know $\boldsymbol{x}_{(-\Delta t)}$ and $\boldsymbol{x}_{(0)}$. The initial conditions $\boldsymbol{x}_{(0)}$ and $\dot{\boldsymbol{x}}_{(0)}$ in Eq. (2) are known. Therefore, we just need to calculate $\boldsymbol{x}_{(-\Delta t)}$ and the step-by-step march can start. At the time station $t=0$, the acceleration $\ddot{\boldsymbol{x}}_{(0)}$ is calculated using Eq. (1), i.e.,

$$\boldsymbol{M}\ddot{\boldsymbol{x}}_{(0)}=\boldsymbol{f}_{(0)}-\boldsymbol{K}\boldsymbol{x}_{(0)} \tag{10}$$

At the time station $t=-\Delta t$, the displacement $\boldsymbol{x}_{(-\Delta t)}$ is approximated using Taylor series expansion at the time station $t=0$,

$$\boldsymbol{x}_{(-\Delta t)}=\boldsymbol{x}_{(0)}-\Delta t\dot{\boldsymbol{x}}_{(0)}+\frac{1}{2}\Delta t^2\ddot{\boldsymbol{x}}_{(0)} \tag{11}$$

Now, $\boldsymbol{x}_{(0)}$ and $\dot{\boldsymbol{x}}_{(0)}$ are known (initial conditions), $\ddot{\boldsymbol{x}}_{(0)}$ is calculated using Eq. (10). $\boldsymbol{x}_{(-\Delta t)}$ can be computed using Eq. (11). Therefore, $\boldsymbol{x}_{(\Delta t)}$ can be calculated using Eq. (9) because both $\boldsymbol{x}_{(-\Delta t)}$ and $\boldsymbol{x}_{(0)}$ are already known. The step-by-step march can start.

3. Explicit computational procedure

When the first derivatives of the damping kernel functions are smooth and continuous, the explicit computational procedure for the non-viscously damped structure systems is summarized as follows.

1.0 Initial calculations

1.1 Compute $\boldsymbol{M}$, $\boldsymbol{K}$, select Δt and calculate matrix $\dot{\boldsymbol{G}}_{(k\Delta t)}$, $(k=0, 1,2, \ldots, n)$

1.2 Solve $\boldsymbol{M}\ddot{\boldsymbol{x}}_{(0)}=\boldsymbol{f}_{(0)}-\boldsymbol{K}\boldsymbol{x}_{(0)} \Rightarrow \ddot{\boldsymbol{x}}_{(0)}$

1.3 Solve $\boldsymbol{x}_{(-\Delta t)}=\boldsymbol{x}_{(0)}-\Delta t\dot{\boldsymbol{x}}_{(0)}+\frac{1}{2}\Delta t^2\ddot{\boldsymbol{x}}_{(0)} \Rightarrow \boldsymbol{x}_{(-\Delta t)}$

1.4 Set $i=0$

2.0 Calculation for each time station

2.1 Solve

$$\boldsymbol{M}\boldsymbol{x}_{[(i+1)\Delta t]}=\left\{\boldsymbol{f}_{(i\Delta t)}-\boldsymbol{K}\boldsymbol{x}_{(i\Delta t)}-\sum_{k=0}^{i}\boldsymbol{G}_{(k\Delta t)}\boldsymbol{x}_{[(i-k)\Delta t]}\right\}\Delta t^2+\boldsymbol{M}\{2\boldsymbol{x}_{(i\Delta t)}-\boldsymbol{x}_{[(i-1)\Delta t]}\}$$

$\Rightarrow \boldsymbol{x}_{[(i+1)\Delta t]}$

2.2 For the next time station. Replace i by $i+1$ and repeat Step 2.1.

4. Examples

In this section, the dynamic responses of MDOF structure system with double exponential model dampers are studied using the previously described method, implicit method proposed by Liu [34], and the method proposed by Adhikari and Wagner [33]. The dynamic responses of SDOF structure system with Gaussian model damper are investigated using both the previously described method and implicit method proposed by Liu [34]. The merits and limitations are also discussed. All the computations are performed on a personal computer with a Windows 7 (Microsoft, Redmond, Washinton) operating system. The computer has an AMD Phenom II×2550 Processor (Advanced Micro Devices, Sunnyvale, California), and the frequency of the CPU is 3.10 GHz. The computer also has 4 GB random-access memory.

4.1. MDOF structure system with double exponential model dampers

The mass matrix of MDOF structure system with N degree of freedom is

$$\boldsymbol{M}=\boldsymbol{I}m_1 \tag{12}$$

where $\boldsymbol{I}$ is a $N\times N$ identity matrix. And $m_1=2$ kg in this case.