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Anti-plane waves near an interface between two piezoelectric half-spaces

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ABSTRACT

We show the existence of certain new waves that can propagate near an interface between two half-spaces of different piezoelectric ceramics, where the interface is modeled by a membrane with the surface/interface elasticity [1]. The current configuration can be reduced into a number of well-known results as special cases, such as Love wave, Bleustein and Gulyaev wave. Together with our previous work for the imperfect interface [2], a full range of consideration of the interface affecting the anti-plane waves is now completed.

1. Introduction

Anti-plane piezoelectric wave near a free surface with exponential decay feature was firstly derived by Bleustein [3] and Gulyaev [4]. The derivation was then extended to the configuration of a wave near an interface between two piezoelectric half-spaces by Maerfeld and Tournois [5]. More recently, Fan et al. [2] showed a set of interface waves by considering a weakened interface described by the so-called spring-model. In the present paper, with consideration of the interface elasticity under the frame work of Gurtin and Murdoch [1], we find a new set of interfacial waves. Together with our previous work [2], the interface waves are classified in a full range condition of the interfaces, which will be summarized in our concluding remarks.

Consider two ceramic half-spaces with an interface. The x_1 and x_3 axes are in the interface where $x_2=0$. $x_2>0$ is occupied by ceramic A and $x_2<0$ by ceramic B. The ceramics are poled along the x_3 or $-x_3$ direction. Consider the so-called anti-plane motions. The displacement vector and the electric potential are given by $u_1=u_2=0$, $u_3=u(x_1, x_2, t)$ and $\phi=\phi(x_1, x_2, t)$. A function, ψ, can be introduced [3] through $\phi=\psi+eu/\varepsilon$, where $e=e_{15}$ and $\varepsilon=\varepsilon_{11}$ are the relevant piezoelectric and dielectric constants. The governing equations for u and ψ in the half-spaces are given by [3]:

$$\begin{aligned} &\bar{c}_A\nabla^2 u_A=\rho_A\ddot{u}_A, \quad \nabla^2\psi_A=0, \quad x_2>0, \\ &\bar{c}_B\nabla^2 u_B=\rho_B\ddot{u}_B, \quad \nabla^2\psi_B=0, \quad x_2<0, \end{aligned} \tag{1}$$

where $\bar{c}=c+e^2/\varepsilon$, and $c=c_{44}$ is the relevant elastic constants. ∇^2 is the two-dimensional Laplacian. The subscripts A and B indicate quantities in ceramic A and ceramic B. For an interface wave, we require all fields to vanish when $x_2\to\pm\infty$. For $x_2>0$, consider the possibility of the following waves propagating in the x_1 direction:

$$\begin{aligned} &u_A=U_A\exp(-\eta_A x_2)\cos(\xi x_1-\omega t), \\ &\psi_A=\Psi_A\exp(-\xi x_2)\cos(\xi x_1-\omega t), \end{aligned} \tag{2}$$

where U_A and Ψ_A are undetermined constants, ω and ξ are the frequency and wave number, and

$$\eta_A^2=\xi^2-\frac{\rho_A\omega^2}{\bar{c}_A}=\xi^2\left(1-\frac{v^2}{v_A^2}\right)>0, \tag{3}$$

where $v=\omega/\xi$ and $v_A^2=\bar{c}_A/\rho_A$. For the continuity conditions, we need explicitly expression of T_{23} and D_2 in ceramic A, denoted by T_A and D_A:

$$\begin{aligned} T_A=\bar{c}_A u_{A,2}+e_A\psi_{A,2}=-[&\bar{c}_A\eta_A U_A\exp(-\eta_A x_2) \\ &+e_A\xi\Psi_A\exp(-\xi x_2)]\cos(\xi x_1-\omega t), \end{aligned} \tag{4}$$

$$D_A=-\varepsilon_A\psi_{A,2}=\varepsilon_A\xi\Psi_A\exp(-\xi x_2)\cos(\xi x_1-\omega t).$$

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Similarly, for $x_2 < 0$, the solution is

$$u_B = U_B \exp(\eta_B x_2)\cos(\xi x_1 - \omega t),$$
$$\psi_B = \Psi_B \exp(\xi x_2)\cos(\xi x_1 - \omega t), \tag{5}$$

$$\eta_B^2 = \xi^2 - \frac{\rho_B \omega^2}{\bar{c}_B} = \xi^2\left(1 - \frac{v^2}{v_B^2}\right) > 0,$$
$$v_B^2 = \frac{\bar{c}_B}{\rho_B}. \tag{6}$$

$$T_B = \bar{c}_B u_{B,2} + e_B \psi_{B,2} = [\bar{c}_B \eta_B U_B \exp(\eta_B x_2) + e_B \xi \Psi_B \exp(\xi x_2)]\cos(\xi x_1 - \omega t),$$
$$D_B = -\varepsilon_B \psi_{B,2} = -\varepsilon_B \xi \Psi_B \exp(\xi x_2)\cos(\xi x_1 - \omega t). \tag{7}$$

The solutions A and B are jointed along the interface. For the interface continuity conditions, we need to consider the interface elasticity before we set up the conditions.

2. Interface mechanics and continuity conditions

The equilibrium conditions on the surface incorporating interface/surface elasticity are given by [1]

$$[T_{\alpha j} n_j] + \Sigma_{\beta\alpha,\beta} = \rho_s \ddot{u}_\alpha \quad \text{(tangential direction)}$$
$$[T_{ij} n_i n_j] = \Sigma_{\alpha\beta}\kappa_{\alpha\beta}, \quad \text{(normal direction)} \tag{8}$$

where α, $\beta = 1$ and 3; Latin letters take 1, 2 and 3; n_i is the unit normal vector to the surface, the bracket $[*]$ denotes the jump of the quantities across the surface, $\Sigma_{\alpha\beta}$ is the surface stress tensor and $\kappa_{\alpha\beta}$ is the curvature tensor of the surface. In addition, the constitutive equations on the isotropic surface are given by [1]

$$\Sigma_{\alpha\beta} = \sigma_0\delta_{\alpha\beta} + (\mu_s - \sigma_0)(u_{\alpha,\beta} + u_{\beta,\alpha}) + (\lambda_s + \sigma_0)u_{\gamma,\gamma}\delta_{\alpha\beta} + \sigma_0 u_{\beta,\alpha} \tag{9}$$

where σ_0 is the surface tension, λ_s and μ_s are the two surface Lame constants in the dimension of N/m.

For the above defined anti-plane problem, Eq. (8) is simplified as

$$\Sigma_{13,1} + (T_{23})^+ - (T_{23})^- = \rho_s \ddot{u}_3, \quad \text{along the interface} \tag{10}$$

Using of Eq. (9) and assuming interface deform together with the half-spaces, we can further express Eqs. (10) into

$$(T_{23})^+ - (T_{23})^- = \rho_s \ddot{u}_3 - \mu_s u_{3,11} \tag{11}$$

An alternative derivation was given by Benveniste and Miloh [6] and Ma et al. [7] for the static case of (11). Eq. (11) represents a simplest attempt to address the interface effect. We noticed that the interface constitutive equations can be in a more sophisticated version. For example, Fang et al. [10] presented a set of piezoelectric interface constitutive equations for their study of nano-particles. Also, a fully nonlinear constitutive interface theory was available [11].

We consider two types of interfaces, namely, (i) an electroded interface and (ii) an unelectroded interface separately below.

First consider the case when the interface is a grounded electrode. We have, at $x_2 = 0$,

$$u_A = u_B,$$
$$T_A - T_B = \rho_s \ddot{u}_A - \mu_s u_{A,11},$$
$$\psi_A + \frac{e_A}{\varepsilon_A}u_A = 0, \tag{12}$$
$$\psi_B + \frac{e_B}{\varepsilon_B}u_B = 0.$$

The second line of the above Eq. (12-2) is the effect of interface elasticity, translated from (11). More precisely, the first term on the right hand-side of (12-2) is the inertia effect, while the second term is the elastic effect. A numerical demonstration in Section 3 will reveal their contributions to the interface wave. Substitution of (2), (4), (5) and (7) into (12) results in a system of linear homogenous equations for U_A, Ψ_A, U_B and Ψ_B. For nontrivial solutions, the determinant of the coefficient matrix has to vanish, which yields the following frequency equation that determines the wave speed v:

$$[\bar{c}_A(\eta_A - \bar{k}_A^2\xi) + \bar{c}_B(\eta_B - \bar{k}_B^2\xi)] = \rho_s\omega^2 - \mu_s\xi^2, \tag{13}$$

where $\bar{k}_A^2 = e_A^2/(\bar{c}_A\varepsilon_A)$ and $\bar{k}_B^2 = e_B^2/(\bar{c}_B\varepsilon_B)$. Clearly, the waves are dispersive. This may be somewhat unexpected as there is no geometric characteristic length in the problem. Mathematically, the dispersion is caused by that the interface condition in (12) involves both u and its first and second order derivatives.

Let us rewrite (13) as

$$\bar{c}_A\left(\left(1 - \frac{v^2}{v_A^2}\right)^{1/2} - \bar{k}_A^2\right) + \bar{c}_B\left(\left(1 - \frac{v^2}{v_B^2}\right)^{1/2} - \bar{k}_B^2\right) = \mu_s\xi\left(\frac{v^2}{v_s^2} - 1\right) \tag{14}$$

where $v_s^2 = \mu_s/\rho_s$.

We will conduct a numerical demonstration of (14) in Section 3, where the gold or aluminum thin membrane joints PZT-5H ceramic half spaces.

To gain some insight into (14), we consider the following special cases:

(i) *No interface elasticity effect.* Eq. (14) with zero right hand side is read as

$$\bar{c}_A\left(1 - \frac{v^2}{v_A^2}\right)^{1/2} + \bar{c}_B\left(1 - \frac{v^2}{v_B^2}\right)^{1/2} = \frac{e_A^2}{\varepsilon_A} + \frac{e_B^2}{\varepsilon_B} \tag{15}$$

which is the velocity equation for the interfaces waves when the bonding is the so-called perfect condition, given by Maerfeld and Tournois [5].

(ii) No piezoelectricity effect: $k^2 = 0$

$$c_A\left(1 - \frac{v^2}{v_A^2}\right)^{1/2} + c_B\left(1 - \frac{v^2}{v_B^2}\right)^{1/2} = \mu_s\xi\left(\frac{v^2}{v_s^2} - 1\right) \tag{16}$$

This is an extended configuration of Love wave. The above presentation indicates the range of the interfacial wave speed as

$$v_s \le v \le v_A$$

and

$$v_s \le v \le v_B. \tag{17}$$

(iii) Let all the material properties of half-space B to be zero, we are considering a half-space coated with a thin electrode.

$$\bar{c}_A\left(\left(1 - \frac{v^2}{v_A^2}\right)^{1/2} - \bar{k}_A^2\right) = \mu_s\xi\left(\frac{v^2}{v_s^2} - 1\right) \tag{18}$$

If we drop the surface effect in Eq. (18), we have

$$\bar{c}_A\left(\left(1 - \frac{v^2}{v_A^2}\right)^{1/2} - \bar{k}_A^2\right) = 0 \tag{19a}$$